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स्वागताध्यक्ष की ओर से

आर० सी० शर्मा
आई० ए० एस० (अव० प्रा०)
कुलपति, गुरुकुल कांगड़ी विश्वविद्यालय हरिद्वार–२४९४०४

मेरे लिये अत्यन्त प्रसन्नता का विषय है कि गुरुकुल कांगड़ी विश्वविद्यालय में "विज्ञान परिषद् ऑफ इन्डिया" का अखिल भारतीय सम्मेलन दिनाँक १०-११ मार्च, १९८९ को होने जा रहा है जिसमें (१ वैदिक गणित, परम्पराएं एवं अनुप्रयोग तथा (२) आधुनिक विज्ञान एवं तकनालाजी में गणित के अनुप्रयोग पर विशेष सिम्पोजियम भी आयोजित किये जा रहे हैं। श्री स्वामी श्रद्धानन्द महाराज को यह उत्कट इच्छा रही है कि यहाँ वैदिक साहित्य, सस्कृत, धर्म, दर्शन और मानविकी के आधुनिक विषयों के साथ विज्ञान की सभी शाखा-प्रशाखाओं का उच्चस्तरीय अध्ययन हो। विज्ञान महाविद्यालय की स्थापना का उद्देश्य भी यही था। आवश्यकता यह समझी गयी कि भारतीय वैज्ञानिकों के विज्ञान सम्बन्धी शोध कार्यो का गहन अध्ययन और मूल्याँकन विश्व-विद्यालय में होना चाहिए। गुरुकुल की अपनी पहचाम भारतीय दृष्टि का विकास करना और विश्व परिप्रेक्ष्य में उस दृष्टि की प्रतिष्ठा करना है। यह गणित सम्मेलन यदि इस कार्य को प्रोत्साहन करेगा तो परिसर में इस अधिवेशन के आयोजित होने की सार्थकता प्रमाणित हो सकेगी।

वेदांग ज्योतिष के साथ जुड़े गणित का भारत में गौरवपूर्ण इतिहास रहा है। आर्च और याजुष ग्रन्थों से लेकर, आर्यभट्ट, भास्कर, ब्रह्मगुप्त, आर्यभट्ट द्वितीय, श्रीधर, श्रीपति, महावीराचार्य, नारायण पण्डित, मुनीश्वर, बापूदेव शास्त्री, सुधाकर द्विवेदी जैसे ग्रन्थकारों के गणित विषयक का कार्य का आंकलन करते हुए रामानुजम जैसे गणितज्ञों का महत्व हम यदि प्रकाशित करते हैं तो वैदिक, बौद्ध और जैन परम्पराओं की विज्ञान सम्बन्धी खोजों का रोचक इतिहास हमें मिल जाता है। अंकगणित, बीजगणित तथा ज्यामिति के क्षेत्र में भारतीय गणित शास्त्र के व्यापक अनुदान का युक्तियुक्त परीक्षण और मूल्याँकन होना चाहिये।

मेरा पूर्ण विश्वास है कि जहाँ आप गणित शास्त्र के आधुनिक विचारकों की निष्पत्तियों और समस्याओं का निदान खोजेंगे वहाँ भारतीय गणित शास्त्र के सही मूल्याँकन के लिये नवयुवकों को प्रेरित भी करेंगे।

इस शुभ अबसर पर परिषद के अध्यक्ष प्रोफेसर जे० एन० कपूर परिषद् के पदाधिकारियों एवं सम्मेलन के सभी प्रतिभागियों का विश्वविद्यालय स्वागत बरता है।

सम्मेलन की निर्विघ्न सम्पन्नता के लिये मेरी हार्दिक शुभ कामनाएँ।

A GOOD BEGINNING

By

DR. VIRENDRA ARORA

Registrar, Gurukula Kangri Vishwavidyalaya

Hardwar- 242904, U.P.

The deciphering of Brahmilipi by Subhas Kak, a renowned cryptologist and Professor of Computer Science at Lousiana State University has confirmed that the Vedic period is some what 6000-4000 B.C. Prof.Kak. through his recent publications, has come out with a surpising fact that paradoxes of modern scince and logic are well traced in Rigveda. This naturally implies that the state of science and technology during Vedic period was no less than what we have today. This fact is further supported by the exactness of measurements described in Brahmilipi.

Aryabhata' s algorithm, called Kuttaka by ancient Indian mathematicians, was meant to solve linear indeterminate equations. Such solutions are fundamental to many computer science applications, and a method called the Chinese Remainder Method is used to find them. Surprisingly Aryabhata' s method is extremely efficient compu, tationally; in fact, as efficient as recent improvements to the Chinese Remainder Method, that were developed during the last 15-20 years.

Indeed, according to Cazori, "Numerical Symbolism, the science of numbers, and algebra attained in India far greater perfection than they had previously reached in Greece..............Both the form and the spirit of the arithmetic and algebra of modern times are essentially Indian".

It is perhaps the most appropriate time for the scientific world to understand and interpret the scientific and technological messages of our traditional system of knowledge and philosophy.

It is indeed, a matter of great pleasure that our Mathematics Department is not only organising a Symposium on Vedic Mathematics and its Traditions but also has initiated research work in our traditional mathematics and astronomy.

I thank the President and Office Bearers of Vijnana Parishad of India for rightly choosing our Vishwavidyalya as the venue for the Frist Annual Confernce whose deliberations are oraganised in the perspective of Vedic Mathematics. Prof S. C. Tyagi and Prof. S. L. Singh deserve full apreciation for organising this Conference, though the travails of successfull organising a conference is better known to them.

We are aware our lapses, and hope that the delegates will bear with us. Once again I extend a warm welcome to you all.

हरिद्वार नगरपालिका अध्यक्ष की ओर से

पारस कुमार जैन वैद्य

हरिद्वार एक अत्यन्त प्राचीन, पवित्र तीर्थ व भारत प्रसिद्ध ऐतिहाहिक नगर है। हरिद्वार का उपनगर कनखल उससे भी पुराना स्थान है जो भगवान शिव की ससुराल है। कनखल वही स्थान है जहां मां पार्वती ने अपने पिता प्रजापति दक्ष के यज्ञ में अपने पति के अपमान पर आत्मदाह किया था। इस नगरी का पुराणों में भी वर्णन है। यह नगर "विज्ञान परिषद ऑफ इन्डिया" के अधिवेशन में पधारे वैज्ञानिकों, विद्वानों एवं शोध छात्रों का स्वागत करता है।

कनखल (हरिद्वार) हिमालय की तलहटी में स्थित होने के कारण वनौषधियों से भरा पूरा है। यही कारण है कि इस नगर में अनेक प्रसिद्ध फार्मेसियों, आयुर्वैदिक औषधियों के मिर्माण में लगी हैं जिनमें कई विदेशों में भी ख्याति अर्जित कर रहीं है। गुरुकुल कांगड़ी फार्मेसी एवं देश रक्षक औषधालय प्रमुख हैं। इस ऐतिहासिक नगरी में हरिद्वार में प्रदेश के प्राचीनतम आयुर्वैदिक कालेज गुरुकुल कांगड़ी तथा ऋषिकुल आयुर्वेदिक कालेज गत ५० वर्षों से जनसेवा में लगे हैं। इसके साथ ही गुरुकुल आयुर्वेद महाविद्यालय भी आयुर्वेद चिकित्सा का ज्ञान मिश्रित चिकित्सा पद्धति प्रदान कर रहा है।

हरिद्वार में प्रत्तेक ६ वर्षो के पश्चात् अर्द्ध कुम्भ तथा १२ वर्षों के अन्तराल पर महाकुम्भ पर्व लगता है जिसमें भारत के कोने–कोने से लगभग एक करोड़ श्रद्धालु मां गंगा के पवित्र जल में स्नान कर पुण्य लाभ करते हैं, यह एक अद्‌भुत समारोह है। जिसमें बिना विज्ञापन व निमंत्रण के जन समूह उमड़ता है ।

हरिद्वार में आने वाले श्रद्धालु भक्तों की सेवा के लिये गंगा सभा, सेवा समिति. भारत स्काउट व गाइड एवं अन्य संस्थायें एक लम्बे समय से कार्यरत है। नगरपालिका हरिद्वार नगर की बिजली, पानी व सफाई व्यवस्था का कार्य वेहतर ढंग से करने का प्रयास कर रही है ।

अंत में एक बार पुनः हम हरिद्वार नगरपालिका की ओर से अधिवेशन में पधारे सभी वैज्ञानिकों और विद्वानों का हार्दिक स्वागत करते हैं ; अभिनन्दन करते हैं तथा अधिवेशन की सफलता के लिये हार्दिक शुभकामना करते हैं। धन्यवाद !

GREETINGS FROM LOCAL SECRETARY

By

S. L. Singh, G. K. University Hardwar–249404, U. P

It is a matter of great privilege for the Gurukula Kangri Vishwavidyalaya to be entrusted with the very pleasant and most exciting task of organising the First Session of the Vijnana Parishad of India. I consider it to be a great honour to the holy city of Hardwar more so because Hardwar is a newly born as a district.

I welcome the delegates and you all. वास्तव में,

"आना तेरा मुबारक, तशरीफ लाने वाले ।
खुशियां मना रहे हैं हम गुरुकुल वाले ।।"
"क्या मुख से विनती करु, लाज लागत मोय ।
तुम जानत अवगुण करु, कैसे भाऊं तोय ।।"
"मेरे महबूब की सूरत तुझसे मिलती जुलती है।
मुहब्बत की मुहब्बत है, इबादत की इबादत है ।।"

Our limitations to make your stay physically comfortable are enormous. We have tried our best to overcome our limitations and frankly rpeaking, depend on your generosity to forgive us for the lapses on our part.

I take opportunity to address our young dilegates. A true scientist looks beyond the boundaries of the nation he belongs to, especially the Indian scientist whose cultural heritage has been to consider the whole world as his family "वसुधैव कुटुम्बकम्". A good scientist can teach mankind to choose to live rather than to be swayed towards eradication for those, who are not scientists nor have scientific temper, are slaves of passion and are likely to misuse their power and the attainments of science. According to Kathopanishad:

"अस्तु अविज्ञानवान भवति अयुक्तेन मनसा सदा ।
तस्य इन्द्रियाणि अवश्यानि दुष्टाश्वा विवसारथे ।।"

(One who is not endowed with science, nor is associated with it in his mind, has his senses and body not in his control like the rogue horse is not in the control of the charioteer.)

Dear friends ! the entire mankind looks towards scientists to have a promise of life.

Dear delegates ! the whole Gurukul family welcomes you once again, Hardwar welcmes you, the Ganges welcomes you, and the Nagadhiraj Himalaya welcomes you. It will be a great sense of satisfaction for me and the organizers if you carry some pleasant memories from here.

और अंत में,आप अपनी असुविधाओं और मेरी कमियों के लिए क्षमा दान दें । जैसे कि:-

"तेरे दरबार की आली निराली शान ये देखी ।
शखावत ख़ुद तेरी गलियों में चक्कर काटती देखी ।।
फैलाया हाथ जिसने तेरे दरबार में आकर ।
तुझे देते नहीं देखा मगर झोली भरी देखी ।।"

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उद्घाटन-भाषण

आचार्य रामप्रसाद वेदालंकार
कार्यकारी कुलपति (एवं प्रो-उप-कुलपति)

माननीय श्री कपूर जी, अध्यक्ष, चन्देल जी मंत्री, अखिल भारतीय विज्ञानपरिषद, आचार्य प्रियव्रत जी वेदवाचस्पति, भूतपूर्व कुलपति, गुरुकुल कांगड़ी विश्वविद्यालय एवं मेरे सहयोगी साथियो, बाहर से पधारे हुए अभ्यागत महानुभावों तथा प्रिय छात्रो, इस युग के महामानव महर्षि दयानन्द की ब्रह्मा से लेकर जैमिनी मुनि पर्यन्त प्रतिपादित वेदानुकूल ज्ञानविज्ञान की शिक्षा के व्यापक प्रचार-प्रसार की परिकल्पना को साकार करने के लिये स्वामी श्रद्धानन्द ने हरिद्वार से ५ मील दूर गंगा के पवित्र तट पर २ मार्च १९०२ में गुरुकुल कांगड़ी की स्थापना की थी। उस समय कौन यह कल्पना कर सकता था कि इस घने जंगल में पूस के छप्परों में प्रारम्भ यह गुरुकुल आगे चलकर न केवल सरस्वती की आराधना का अपितु सामाजिक एवं राष्ट्रिय जागरण का अग्रदूत बन सकेगा। अपने शैशव काल से ही गुरुकुल ने राष्ट्रीय जीवन पर अपनी अमिट छाप अंकित कर दी। यह संस्था स्वतन्त्रा आन्दोलन, पत्रकारिता, आयुर्वेद, साहित्य, विज्ञान, इतिहास, आदि का प्रेरणा केन्द्र बन गया। उस समम के सर्वश्रेष्ठ ख्याति प्राप्त नेता यहाँ पर आये और गुरुकुल के पवित्र जीवन को देखकर भाव-विभोर हो गये। इसी पावन भूमि में स्वामी श्रद्धानन्द ने बड़े आदर और प्रेम से मि. गांधी को महात्मा गांधी कहकर संबोधित किया था। गुरुकुल का यश चारों दिशाओं में फैलता गया। डा. राजेन्द्र प्रसाद, श्रीमती इन्द्रागांधी, राष्ट्रकवि दिनकर, मुंशी प्रेमचन्द जैसे अनेक राष्ट्रिय नेताओं व साहित्यकारों ने समय-२ पर यहाँ पधार कर स्वंम को गौरवान्वित समझा। प्रारम्भ में यह गुरुकुल २२-२४ बर्ष तक शिवालिक की तराई में गंगा तट पर अपनी कीर्ति पताका फहराता रहा। गुरुकुल शिक्षा के क्षेत्र में प्रारम्भ से उन्नति के शीर्ष पर रहा है, शिक्षा जगत् में इसकी अपनी एक विशिष्ट छवि रही है, यही कारण है कि विश्वविद्यालय अनुदान आयोग की अनुकूल संस्तुति पर भारत सरकार ने १९६२ में गुरुकुल को विश्वविद्यालय की मान्यता प्रदान की। और यहाँ की सभी उपाधियाँ देश के अन्य विश्वविद्यालयों तथा केन्द्रीय एवं प्रान्तीय सरकारों द्वारा अन्य विश्व-विद्यालयों के समान मान्यता प्राप्त हो गयी। विश्वविद्यालय के अन्तर्गत वेद महा-विद्यालय कला, महाविद्यालय, विज्ञान महाविद्यालय, आयुर्वेद महाविद्यालय तथा

देहरादून में कन्या गुरुकुल महाविद्यालय शिक्षा के व्यापक प्रचार प्रसार में लगे रहें । मध्यकाल में उत्तर प्रदेश शासन ने आयुर्वेद महाविद्यालय को अपने अधीन कर लिया । आज भी वहाँ उच्चस्तरीय अध्ययन चल रहा है । इस बिश्वविद्यालय में स्नातक परीक्षोत्तीर्ण छात्र विद्यालंकार, वेदालकार, उपाधियों से विभूष्ति किये जाते हैं, साथ ही वेद, दर्शन,सस्कृत, भारतीय सस्कृति एवं प्राचीनेतिहास, अंग्रेजी, हिन्दी, मनोविज्ञान आदि विषयों में द्विवर्षीय स्नातकोत्तर पाठ्यक्रम के अतिरिक्त उच्चतर अध्ययन एवं अनुसन्धान हेतु पी-एच० डी० शोधोपाधि प्राप्त करने की व्यवस्था भी यहां की गयी है। विज्ञान महाविद्यालय में गणित, भौतिकी, रसायन-शास्त्र, जन्तुविज्ञान, वनस्पति विज्ञान आदि की उच्चस्तरीय शिक्षा के साथ कम्प्यूटर साइस की शिक्षा भी प्रारम्भ हो चुकी है । वैदिक प्रयोगशाला, योग डिप्लोमा, एवं कर्मकाण्ड ड़िप्लोमा इस विश्व-विद्यालय की नवीनतम उपलब्धियाँ हैं । यहाँ के प्राध्यापक एवं छात्र वैदिक प्रयोग-शाला में अनुसन्धान कार्य में लगे रहते हैं । गुरुकुल पत्रिका, प्रहलाद् वैदिक-पथ आदि विभिन्न शोध-पत्रिकाएं दूर–दूर तक ज्ञान–विज्ञान के प्रचार में रंलग्न हैं । समय-२पर शोध–संगोष्ठियों का समायोजन करके उनमें राष्ट्रिय एवं अन्तर्राष्ट्रिय ख्याति प्राप्त विद्वानों को कुलभूमि में आमन्त्रित किया जाता है। जिससे प्राचीन एवं अर्वाचीन के मधुर समन्वय मे राष्ट्रीय-जीवन को महिमा-मण्डित करने का अखण्ड प्रयास निरंतर चलता रहता है । इस सस्था में कार्यरत अधिकारियों एवं शिक्षकों के मस्तिष्क में सदैव राष्ट्र कल्याणकारी शिव-सकल्प जागते रहते हैं । यहाँ के शिक्षक सतत इस चिन्तन मैं संलग्न रहते हैं कि किस प्रकार समाज के सभी वर्ग इस तीर्थ में स्नान करते रहें , इसी सद्भावना के स थ आज भारतीय विज्ञान परिषद् द्वारा हमारे इस गुरुकुल विश्वविद्यालय में समायोजित परिषद् का प्रथम वार्षिक अधिवेशन तथा Vedic Mathematics संगोष्ठी के शुभ अवसर पर आप सबका पुनः हार्दिक स्वागत एवं अभिनन्दन करता हुआ अधिवेशनतथा संगोष्ठी का शुभारम्भ करता हूं ।

Proc. VPI. Vol. 1, 1989

VIJNANA PARISHAD OF INDIA
FIRST ANNUAL CONFERENCE , 1989
GURUKUL KANGRI VISHWAVIDYALAYA , HARDWAR
PRESIDENTIAL ADDRESS

By

Professor J.N. Kapur, Ph.D. ; F.N.A.Sc. ; F.A.SC. ; F.N.A.

At the out set , I must express my sincere thanks to all members of "Vijnana Parishad of India" for the honour they have done me by asking me to be the first President of the Parishad. I must also thank the authorities of Gurukul Kangri Vishwavidyalaya for inviting the Parishad to hold its First Annual Conference in this holy city of Hardwar.

Along with this conference , we shall also be holding a National Symposium on "VEDIC MATHEMATICS, TRADITIONS AND APPLICATIONS". This is appropriate, since this university has many great scholars of Sanskrit and Vedas and as such this Symposium should provide a unique opportunity for meaningful interaction between mathematicians and Vedic scholars , so that we can know more about the truth regarding the origins of the great traditions of Mathematics in India,

India was certainly a world leader in Mathematics , till at least the twelth century and the foundations of this greatness were laid by the great rishies and Vedic scholars who thought about great truths about numbers and geometrical forms in deep forests and in the high ranges of the Himalyas and in holy places like Hardwar . These rishies have not only left us mathematics of which we can be proud, they have also left us great traditions of dedication to learning and scholarship. They were great worshipers of Sarswati and the mathematicians of modern India can draw a great inspiration fron these

great seers who pursued mathematical studies vigrously and unrelentingly for the sheer joy it gave them.
Vedic Mathematics means Mathematics in the Vedas. However recently Vedic Mathematics has come to be associated with the mathematics contained in the book with the title written by Jagatguru Shankaracharyaji Maharaj.

The sixteen sutras or aphorisms carrying out mathematical computations given in thish book have caught the imagination of hundreds of thousands of people in India and abroad. There has been a great enthusiasm for these formulae among the laymen everywhere.

While it is good to be enthusiastic, we should not be swept off our feet by it. We should remeber some basic facts:

(1) The book "Vedic Mathematics" does not deal with "Mathematics in the Vedas" for which there are many other books. It deals with only sixteen Ganita sutras which Swamiji stated were contained in an Appendix of Atharvaveda.

(ii) This Appendix is not found in any of the various of the Vedas availble with us inspite of deliberate search for these. I am sure the great scholars of the Vedas in this university will be able to tell us if there is anything resembling these in the Vedas.

(iii) When Professor K.S. Shukla, a very distinguished scholar of ancient Indian Mathematics took a copy of Atharveda to Swamiji personally and asked him to show where in it, these sutras were contained, he said: These are contained in a copy of Atharva Veda which I have but he did not produce this copy inspite of Dr. Shukla's request.

(iv) In an Address to university students in California, Swamiji stated that these formulae were in one place in the body of the Atharvaveda under the title of Ganita sutras' and his contribution was to interpret these properly with the help of ancient lexigraphies and by deep mediation in forests. However the term 'Ganita' was not used in the Vadas and was used only later by Jain scholars.

(v) Swamiji's desciple Manjula Trivedi stated that these sutras where not contained in the vedas as such, but were constructed from material's scattered in the vedas here and there. This Contradicts Swamiji's own statement.

(vi) As Professor Shukla has demonstrated, most of the ideas contained in these sutras were developed as late as eighteenth century and are not all based on ideas otherwise known to be contained in vedas. Swamiji was familiar with all this mathematics because he had done M.A. in mathematics. We do not expect a great charm the mathematcal ideas contained in the rest of vedas and ideas contained in the sixteen sutras also said to be contined in the vedas.

(vii) The language of these sutras is quite different from vedas Sanskrit. Again the vedic scholars here can throw light on the aspect.

(viii) It is said that whether 'Vedic Mathematics' belongs to Vedas or not may be controversial, but that is good mathematics can not be denied. It is interesting mathematics, it is enjoyable mathematics, it is recreational mathematics but it is elementary mathematics.

(ix) These sutras simplify calculations and simplify considerably for some problems, for other problems the calculations may even become longer. As such these cannot be used to replace our standard methods of calculation.

(x) Though in some situations these can give very quick answers, again these cannot compete with calculators in finding 89^{89}. *ln* 89, sin 89 etc.

(xi) There is no doubt that both children and adults enjoy these short cuts tremendously. These short cuts are obtained by a deep understanding of the place-value system. What these demonstrate is that mathematics can be best understood by understanding its structure.

(xii) What this book demonstrates is that mathematics can greatly enjoyable if it is presented properly. Swamiji was a great master in explaining mathematical matters lucidly, clearly and in a fascinating manner.

(xiii) This book also demonstrates the great intellectual joy which mathematics alone can give. This book represents a very very small part of mathematics. If this small part can give so much enjoyment, one can expect wonderful thrils from mathematics, provided it is explained and understand properly.

(xiv) These opinions about the book 'Vedic Mathematics' are my own and do not represent the views of the parishad. In fact the Parisad has organised this symposium to know the Truth and Truth nomes when there is a free expression of opinions by all.

One object of the Parishad is to make possible the 'Enjoyment of Mathematics' by millions of our countrymen. Mathematics can

prosper only if millions of people enjoy it and love it. Today the situation is just the opposite. Due to disastrous teaching of mathematics in our schools, millions have developed a dislike for mathematics they suffer from mathphobia. They respect it but they believe that mathematics is not their cup of tea.

We have to revers this state of affairs. We do not want just respect for mathematics, we want love for mathematics we are extremely greatful to Swamiji for demonstrating that mathematics can be enjoyed. We want hundreds of missionaries like Swamiji to go out and spread the message of fascination of Mathematics. I have myself written extensively on this subject.. Two hundred of my articles are included in the eight volumes of my book, Fascinating World of Mathematical Sciences, published by Mathematical Sciences Trust Society, New Delhi.

Tte major objective of the Parishad is to work for Propagating Applications of Mathematics in all walks of life. We would like to promote Applications of Mathematics in curricula of primary schools, secondary schools,undergraduate colleges and postgraduate departments.

Our students are not learning sufficient applications of mathematics to motivate them towards learning of mathematics. Accordingly a few years ago UNESCO commissioned me to write a book on these applications, using only secondary school mathematics, for the use of future teachers of Mathematics under preservice training in training colleges, I accordingly wrote the book' Insight into Mathematical Modelling' which was published by Indian National Science Academy. This book gives about one hundred applications of mathematics in astronomy geography, physics, physiology, management finance etc.

At the undergraduate level, we want to have a compulsory course in Statics, Dynamics and Hydrostatics and this used to account for one-third of the undergraduate course. Over the years this course has been diluted or entirely replaced by courses on pure mathematics, so much so that a student can get B.Sc. degree without kdowing ane single application of mathematics. This is similar to the situation that occured eralier in other parts of world. To remedy this, undergraduate courses in mathematical modelling were introduced there. These are are also being introduced in our country and I have written a book on 'Mathematical Modelling' for this purpose which has been published by Wiley Eastern, New Delhi and John Wiley, New York. This gives aqplictions of ordinary differential equations, difference equations, pratal differential equations, integrodifferential equations, graph theory, optimization techniques etc. to problems in physics, economic, biology, astronomy, environment control, population control, epidemics, genetics etc.

At the postgraduate level, courses on spherical Astronomy, Hydrodynamics and Electromagnetic Theory used to be popular. Later new courses in mathematical Statistics and operations research was introduced. More recently courses on Biomathematics have been introduced. I have written a book on 'Mathematical Models in Biology and Medicine' and another on 'Maximum Entropy Models in Science and Engineering' which can be used at this level.

Our main problems however are at the research level. Mathematics in India has not played the role it should have played in helping in solving problems of national life viz. Problems which arise in CSIR, DRDO, ISRO and other laboratories, problems which our industry faces, problems which large scale use of computers has

introduced, problems of pollution and population control, problems of modern engineering like pattern recognition, artificial intelligence, robotics etc. , problems of atmospheric and ocean sciences and so on.

Our universities have failed to poduce real industrial and applied mathematicians. We have prodused enough Ph.D.'s in academic applied mathematics. However we have produced very little real life applied mathematics.

The goals of the Parishad for the next few years are clear.

(i) To work for popularisation of mathematics among students and public.

(ii) To work for introduction of more 'applications of mathematics in curricula at all levels.

(iii) To work for greater interaction of mathematics and users of mathematicis for the solution of those national problems which require the use of mathematics.

(iv) To intensify training and research in Industrial and applied mathematics.

If we can use mathematics maningfully, we can save hundreds of crores of rupees for the country.

There are nany challenges for mathematieians in the modern world. I am sure that members of the Parishad will accept the challenge of placing India on the world map of Industrial and Applied Mathematics.

The Parishad seeks the co-operation of all of you in this task.

I hope we shall have very meningful and exiting discussion in the conference and the conference will give us goals for which we shall work in the next years ahead.

Jai Ganit Jai Hind.

Proc. VPI., Vol. 1, 1989

FIRST ANNUAL CONFERENCE OF VIJNANA PARISHAD OF INDIA

GURUKUL KANGRI UNIVERSITY, HARDWAR, March 10-11, 1989

AN INTRODUCTION OF "VIJNANA PARISHAD OF INDIA"

(Society for Applications of Mathematics)

By

R. C. Singh Chandel

Secretary, Vijnana Parishad of India, D. V. Postgraduate College Orai-285001, U. P., India

Honourable Vice-Chancellor, G. K. University Hardwar, Principal Tyagiji, President Prosessor Kapur, Local Secretary Professor S. L. Singh, Delegates and Guests:

I am personally grateful to this assembly which has given me an oportunity to speak and introduce with the "VIJNANA PARISHAD OF INDIA", which is a society dedicated to Applications of Mathematics in all fields of Science and Technology and as such it has a crucial role to play because it is only the society in India explicitely devoted to this goal.

In 1970, Dr. B B. Lal the Ex-Principal, D. V. Postgraduate College., Orai 285001, U. P. desired of his science teachers to start a publication of a Scientific Research Journal from the College, containing a treasure of quotable references for the help and guidance of research scholars and worth preservable by the Libraries. Responding to his inspiration, Dr. R.C. Singh Chandel of Mathematics Department came forward to fulfil his dream and to facilitate the research activities and encourage the young researchers by taking of the publication of Mathematical Research Journal "JÑĀNĀBHA" ज्ञानाभ

(Ray of Knowledge) published by "VIJNANA PARISHAD" of D. V. Postgaduate College, Orai. It was started in 1970-71, with generous co-operation of various experts as active members of its Editorial Board, renowned Professor J. N. Kapur as its Chief Advisor, Dr. B. B. Lal as Patron and R. C. Singh Chandel as Editor.

As soon as the Journal "JÑĀNĀBHA" saw the light of the day in 1971 it attracted the attention of the scholars of the World and crossed the boundary os the Country in no time. As it could not be kept limited to the College, the "VIJNANA PARISHAD'. of D. V. Postgraduate Colleg, Orai adopted slightly modified name "VIJNANA PARISHAD OF INDIA" in place of "VIJNANA PARISHAD", indicating an extension of its field to other countries, and besides, having an inependent status of its own free from the control of an institution. Here we can not forget the good name of Professor H. M. Srivastava (University of Victoria, Canada), who in 1972 was invited to work as a Foreign Secretary of the PARISHAD and who gladly agreed to work as one of the Editors. Some foreign experts of different fields of Mathematics and other Sciences were also included in the Editorial Board,

Since the begining of 1972 "VIJNANA PARISHAD OF INDIA" has Reciprocity Agreement with "AMERICAN MATHEMATICAL SOCIETY". The Parishad is one of the following four Indian Societies which have mutual "RECIPROCITY AGREEMENT" with "AMERICAN MATHEMATICAL SOCIETY": 1. Allahabad Mathematical Society 2. Calcutta Mathematical Society 3. Indian Mathematical Society and 4. Vijnana Parishad of India.

The Mathematical content of the Journal "JÑĀNĀBHA" of "VIJNANA PARISHAD OF INDIA" is indexed in the American

Biweekly "CURRENT MATHMATICAL PUBLICATIONS" and is reviewed among oshers, by Mathematical Reviews (U. S. A.), Zetralblatt fur Mathematik (W. Germany) and Applied Mechanics Reviews (U. S. A.).

Upto 1985, the Parishad was an unregistered body and its aim was only to publish regularly the Research Journal "JÑĀNĀBHA". On 24th August, 1986 under the Presidentship of Professor J. N. Kapur a meeting was held to I. I. T. Kanpur and it was decided to get it registered. Consequently it was got registered on 4th July 1988 with D. V. Postgraduate College, Orai-285001, U. P. as its registered office having the following aims and activities:

AMIS OF THE PARISHAD

To work for the development of Mathematical Sciences in India. To encourage the interaction of Mathcmasics with Engineering, Industrial Medical, Social, Physical, Biological and Computer Sciences, and specially

1. To work for the popularisation of Mathematics among students and public.
2. To work for introduction of the more "Applications of Mathematics" in curricula at all levels.
3. To work for greater interaction of Mathematics and users of Mathematics for the solutions of those national proplems which require the use of Mathematics.
4. To intensify training and research in Industrial and Applied Mathematics.

ACTIVITIES

1. To publish Research Journals and Monographs in Mathematical Sciences.

2. To publish Journals for popularising Mathematical Sciences and their teaching techniques.
3. To hold conferences, Symposia, Exhibitions and Fairs for the development of Mathematical Sciences.
4. To establish Libraries for Research Scholars.
5 To encourage Mathematics Education
6. To publish abvance books.
7. To encourage the interdiciplinary activities
8. To establish Local Chapters of the Parishad in different Universities.
9. To award prizes to the talented students
10. To award prizes and honours to the teachers and researchers of India for their unique work of eminence.
11. To take other steps considered necessary to promote the aims of the Parishad.

At present Parishad has 65 Life Members including 22 Foreign members. Parishad gets 18 Journals in Exchange of JÑĀNĀBHA including 8 Foreign Journals.

It could not be out of place to mention here that since the very inception of the first publication of the Journal JÑĀNĀBHA or in other words since last 19 years we are carrying on our mission by our own resources and have received no financial aid from the Government· or from any other agency. Further it is a matter of great satisfaction and appreciation of the Parishad that the Journal is being published regularly with poor financial resources and only scanty help from voracious readers, researchers and authors. Every one here can realize the difficulties which our Parishad has been facing for the publication of the Journal and fulfilling the aims of the Parishad. No organization or Association can run without monetory funds. Our Mathmatical progress may too hemper for want of necessary funds. As such the

Parishad has to look to the perpetual resources from which we may continue to fulfil the aims of the Parishad. In case we happen stand care free in the finacial domain for regular publication to the Journal and execution of other activities aimed at by the Parishad, we can carve out an important place for it in the Mathematical World, and the torch of our mission may give light to other countries too.

Here we are holding First Annual Conference of "VIJNANA PARISHAD OF INDIA" under the auspices of Gurukul Kangri University Hardwar, partially sponsored by DRDO, New Delhi. As a part of the Conference, we are also organising a NATIONAL SYMPOSIUM on " VEDIC MATHEMATICS, APLICATIONS AND TRADITIONS" for which we are very much thankful to the Honourable Vice-Chancellor Shri R. C. Sharma, Local Secretary Professor S. L. Singh, Principal Tyagiji and the Registrar Dr. Arora. This is only due to their keen interest that we could get together here at a very short notice.

I thank you once again the brothers and sisters assembled here from different parts of the country and welcome them, and also seek their full co-operation and good wishes for the success of the conference.

Jai Parishad

Proc. VPI., Vol.1, 1989

REPORT ON FIRST ANNUAL CONFERENCE OF "VIJNANA PARISHAD OF INDIA"

(March 10-11, 1989)

VENUE: Gurukul Kangri University, Hardwar, U. P.

By

R. C. Singh Chandel

Secretary, Vijnana Parishad of India, D. V. Postgraduate College Orai–285001, U. P., India

First Annual Conference of "VIJNANA PARISHAD OF INDIA was held at Hardwar under the auspices of Gurukul Kangri University Hardwar on March 10–11, 1989 partially supported by DRDO, New Delhi. The Conference was presided by Professor J. N. Kapur (Delhi) and inaugrated by the Vice-Chancellor, Gurukul Kangri University, Hardwar. From all parts of the country, 67 delegates participated in the conference out of which the following were the distinguished persons of Mathematics:

Professor J.N. Kapur (Delhi), S. N.Pandey (Roorkee), Chandra Mohan (Roorkee), Krishna Lal (Varanasi), R. N. Kaul (Delhi). Aruna Kapoor (Delhi), V. P. Saxena (Gwalior), S. P. Arya (Delhi), Narendra Puri (Roorkee), R. S. Lal (Siwan), J. M. C. Joshi (Nainital), V. N. Sharma (Rampur), R. C. Singh Chandel (Orai), S. Lal (Agra) R. S. Tripathi (Gorakhpur) and P. R. Subramanian (Madras).

During the Conference of two days, 15 invited talks were given while 35 papers were presented.

In his inaugral speach, the Vice-Chancellor of Gurukul Kangri University, Hardwar exhorted that keeping this University venue of the Conference, which was founded for the propagation of Vedas the Conference will be quite successful if in output we are able to use

our Vedas in the progress of Modern Science and Technology through the "VIJNANA PARISHAD OF INDIA".

The Programme included Presidential address (General as well as Technical) by Professor J. N. Kupur besides other key note addresses, invited talks and paper reading sessions: specially one for National Symposium on "MATHEMATICS, APPLICATIONS AND TRADITIONS" and other for "APPLICATIONS OF MATHEMATICS IN MODERN SCIENCE AND TECHNOLOGY". The focal theme of the Conference was to discuss VEDIC MATHEMATICS and to present newer areas of use of Mathematics in Engineering, Physical, Biological, Social, Medical and Computer Sciences.

"Vedic Mathematics" was hot topic of the Conference. Dr. Naredra Puri (Roorkee) in his talk on "OVERVIEW OF VEDIC MATHEMATICS" presented some "Sutras" in Sanskirat and make their applieations in solving some particular problems of Arithmetic and claimed that sutras so used were picked up from Vedas. On the other hand some speakers like Professor Kapur and Dr. Shashi Prabha Arya expressed the view that the Mathematics particularly so called "VEDIC MATHEMATICS" was not present in any of the Vedas. This fact can be visualized by the Sanskirat language itself used in "VEDIC MATHEMATICS" written by Shankaracharya. This is different from the Sanskirat used in Vedas. Then the supporters claimed that it was present in those portions of the Vedas which have since lost. Then it was the final consensus that the so called "VEDIC MATHEMATICS" by Shankracharya is not "VEDIC MATHEMATICS". Actually, it is Ancient Mathematics which may be called "HINDU MATHEMATICS". Seeing that these sutras no doubt we may make fast calculations of some particular problems of Arithmetic very easily, it was recommended to make further researches to get applications of these

sutras in Higher Mathematics and in Modern Science and Technology so that India may bring again her name on the international map and can guide again the World as a *Jagadguru* in the field of Science and Technology

Professor J. N. Kapur in his Presidential Address stated that "Vijnana Parishad of India" is a society dedicated to Applications of Mathematics in all fields of Science and Technology and as such it has a crucial role to play because it is the only society in India explicitely devoted to this goal. He pointed out the great job opportunities for properly trained mathematical scientists. He wanted all mathematicians to come forward to play a leadership role in the computer revolution. He wanted all Mathematics Departments to be epuipped with computer laboratories and all curricula to take into account modern applictions and computers.

In the Symposium on "VEDIC MATHEMATICS", he expressed his views that "Vedic Mathematics" by Jagadguru Shankaracharya did not deal with mathematics of the Vedic times, its level is that of Intermediate level, but it can have a great role as enrichment material to remove mathematical anxiety from the students, of course it cannot compete with computers which are million times faster.

In his velidictory address. he invited every one to have intense love for the fascinating and exciting subject of Mathematics and to work hard to learn and teach new areas of applications of Mathematics.

Professor Kapur also inspired the delegates coming from all parts of the country to execute the recommendations of the Conference in their Institutions and to further the atmosphere of researches of mathematics and their Applications in Science and Technology.

The conference came to a close with vote of thanks to the University authorities particularly the learned Vice-Chancellor Shri R. C. Sharma "Lions" of Rishikesh, Local organising Secretary Professor S. L. Singh Conveners and members of different Committees making the Conference a success.

Income and Expenditure Account for the Year Ended 31st March, 1974

Income	Amount Rs.	Excpenditure	Amount Rs.
Opening Balance	3156.70	Postage	44.00
By Subscription and Publication Charges	1289.58		
		Total	44.00
		Closing Balance	4402.28
	4446.28		4446.28

(Sd.) B. B. Lal
Patron and Principal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary and Treasurer

Income and Expenditure Account for the Year Ended 31st March, 1975

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	4402.28	Postage	513.95
By Subscription and Publication Charges	3975.22	Publication	4875.09
		T.A.	260.90
		Miscellaneous	134.20
		Total	5784.14
		Closing Balance	2593.36
	8377.50		8377.50

(Sd.) B. B. Lal
Patron and Principal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary and Treasurer

Inceme and Expenditure Account for the Year Ended 31st March, 1976

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	2593.36	Postage	567.75
By Subscription and Publication Charges	8363.20	Pblication	3547.19
		T. A.	31.96
		Miscellaneous	31.80
		Total	4178.70
		Closing Balance	6777.86
	10956.56		10956.56

(Sd.) B. B. Lal
Patron and Principal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary and Treasurer

Income and Expenditure Account for the Year Ended 31st March, 1977

Incom e	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	6777.86	Postage	361.60
By Subscription and Pubtication Charges	4463.31	Closing Balance	10879.57
	11241.17		11241.17

(Sd.) B. B. Lal
Patron and Principal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary and Treasurer

Income and Expenditure Account for the Year Ended 31st March, 1978

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	10879.57	Postage	581.60
By Subscription and publication Charges	1934.55	Publication	5420.00
		T. A.	24.66
		Total	6026.26
		Closing Balance	6787.86
	12814.12		12814.12

(Sd.) B. B. Lal
Patron and Principal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary and Treasurer

Income and Expenditure Account for the Year Ended 31st March, 1979

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	6787.86	Postage	451.05
		Publication	1000.00
By Subscription and Publication Charges	1170.99	T. A.	60.38
		Miscellaneous	28.10
		Total	1539.53
		Closing Balance	6419.32
	7958.85		7958.85

(Sd.) Dr. B. B. Lal
Patron and Principal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R C Singh Chandel
Secretary and Treasurer

Income and Expenditure Account of the Year Ended 31st March, 1980

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	6419.32	Postage	97.30
By Subscription and	4478.30	Publication	5500.00
Publication Charges etc.		T. A.	80.37
		Total	5677.67
		Closing Balance	5219.95
	10897.62		10897.62

(Sd.) B. B. Lal
Patron and Pirncipal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary and Treasurer

Income and Expenditure Account of the Year Ended 31st March, 1981

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	5219.95	Postage	584.65
By Subscription and	4456.14	Publication	1000.00
Pubtication Charges etc.		S. A.	110.76
		Total	1695.41
		Closing Balance	7980.68
	9676.09		9676.09

(Sd.) B. B. Lal
Patron and Principal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandcl
Secretary and Treasurer

Income and Expenditure Account for the Year Ended 31st March, 1982

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	7980.68	Postage	573.70
By Subscription and Publication Charges	2501.03	Publication	2000.00
		T. A.	80.56
		Miscellaneous	15.00
		Total	2669.26
		Closing Balance	7812.45
	10481.71		10481.71

(Sd.) B. B. Lal, President — (Sd.) Tilak Singh, Society Auditor — (Sd.) R. C. Singh Chandel, Secretary–Treasurer

Income and Expenditure Account for the Year Ended 31st March, 1983

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	7812.45	Postage	550.65
By Subscription, Publication Charges and loan of Rs. 725.00 given by R C.S. Chandel on 1-1-83	9333.89	Publication	15078.15
		Miscellaneous (Refund of loan)	725.00
		Total	16353.80
		Closing Balance	792.54
	17146.34		17146.34

(Sd.) B. B. Lal, President — (Sd.) Tilak Singh, Society Auditor — (Sd.) R. C. Singh Chandel, Secretary–Treasurer

Income and Expenditure Account for the Year Ended 31st March, 1984

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	792.54	Postage	1682.25
By Subscription and Publication Charges	9057.10	Publication	5845.00
		Stationery	72.50
		Miscellaneous	234 40
		Total	7834.15
		Closing Balance	2015.49
	9849.64		9849.64

(Sd.) B. B. Lal	(Sd.) Tilak Singh	(Sd.) R. C. Singh Chandel
President	Society Auditor	Secretary-Treasurer

Income and Expenditure Account of the Year Ended 31st March, 1985

Income	Amount Rs.	Expenditure	Amount Rs.
Opennig Balance	2015.49	Postage	891.75
By Subscription, Pubtication Charges and Loan of Rs . 1500.00 given by R. C .S Chandel	8215.78	Publication	7334.00
		Total	8225.75
		Closing Balance	2005.52
	10231.27		10231.27

(Sd.) B. B. Lal	(Sd.) Tilak Singh	(Sd.) R. C. Singh Chandel
President	Society Auditor	Secretary-Treasurer

Income and Expenditure Account for the Year Ended 31st March, 1986

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	2005.52	Postage	1311.00
By Subscription and Publication Charges	14361.17	Publication	9800.00
		Loan refund	1500.00
		Miscellaneous	5.00
		Total	12616.00
		Closing Balance	3750.69
	16366.69		16366.69

(Sd.) J. N. Kapur, President   (Sd.) Tilak Singh, Society Auditor   (Sd.) R. C. Singh Chandel, Secretary–Treasurer

(Sd.) M. M. Lal & Company, C. A., Kanpur

Income and Expenditure Account for the Year Ended 31st March, 1987

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	3750.69	Postage	1449.90
By Subscription and Publication Charges	11300.23	Publicaton	13000.00
		Total	14449.90
		Closing Balance	601.02
	15050.92		15050.92

(Sd.) J. N. Kapur, President   (Sd.) Tilak Singh, Society Auditor   (Sd.) R. C. Singh Chandel, Secretary –Treasurer

(Sd.) M. M. Lal & Company, C. A., Kanpur

Income and Expenditure Account for the Year Ended 31st March, 1988

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	601.02	Publication	5011.00
By Subscription and Publication Charges	4923.82		
		Closing Balance	513.84
	5524.84		5524.84

(Sd.) J. N. Kapur, President (Sd.) Tilak Singh, Society Auditor (Sd.) R. C. Singh Chandel, Secretary–Treasurer

(Sd.) M. M. Lal & Company, C. A., Kanpur

Income and Expenditure Account for the Year Ended 31st March, 1989

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	513.84	Postage	4930.90
By Subscription and Publication Charges	23147.51	Publication	5988.75
		Conference	8326.00
		Stationery	218.00
		Miscellaneous	15.00
		Total	19478.65
		Closing Balance	4182.70
	23661.35		23661.35

(Sd.) J. N. Kapur, President (Sd.) Tilak Singh, Society Auditor (Sd.) R. C. Singh Chandel, Secretary –Treasurer

(Sd.) M. M. Lal & Company, C. A., Kanpur

LIST OF PARTICIPANTS WITH THE TITLES OF THEIR TALKS/PAPERS PRESENTED IN THE FIRST ANNUAL CONFERENCE OF " VIJNANA PARISHAD OF INDIA " HELD AT HARDWAR ON MARCH 10-11, 1989.

1. Professor J. N. Kapur, C–766, New Friends Colony, New Delhi
   President, Vijnana Parishad of India,
   Presidential Address, General and Technical; and Valedictory Address.
2. Dr. R. C. Singh Chandel, D. V. Postgraduate, College, Orai-285001, U. P.
   Secretary, Vijnana Parishad of India,
   Secretorial Report and a talk on
   "Binomial analogues of addition theorems of Srivastava, Lavoie and Tremblay and their applications''
3. Dr. Narendra Puri, Department of Civil Engineering, Roorkee University, Roorkee, U. P.
   "Overview of Vedic Mathematics''
4. Professor S. N. Pandey, Roorkee Universiry, Roorkee, U. P.
   "Gravitational significance of some hyper surfaces imbeded in a 5–flat''.
5. Professor, R. N. kaul, Delhi University, Delhi
   "Steiner's problems and mathematical programming"
6. Professor Chandra Mohan, Roorkee University, Rookee, U. P.
   "Computational aspects of non-linear optimization problems and their practical applications''.
7. Professor V. P. Saxena, Jiwaji University, Gwalior, M. P.,
   "Modern Trends in Bio-Mathematics"
8. Professor K. Lal, B. H. U., Varanasi, U. P.
   "Mathematics and its applications to Technology"
9. Professor Aruna Kapoor, Jamia Millia Islamia, Delhi "Applicaiions of Mathematics to certain problems of Duct Acoustics"
10. Professor J. M. C. Joshi, Kumayun University, Nainital, U. P.
    "S. R. RAMANUJAN – A Historical Perspective''
11. Dr. Shashi Prabha Arya, Maitrayi College, New Delhi
    "Mathematics in Vedic Literature''

12. Professor R. S. Lal, D. A. V. College, Siwan, Bihar
"A novel method of factorization by Vedic Mathematics"
13. Dr. R. S. Tripathi, Gorakhpur University, Gorakhpur, "Developments of numbers in Vedic Age and India-Russia relations"
14. Dr. S. Lal, Agra University, Agra "Some examples in Algebra"
15. Dr. V. N. Sharma, Govt. Raja P. G. College, Rampur-244901,
"The factor 10^{27} in Hindu numeration and indeterminancy principle"
16. Professor P. R. Subramanyan, Department of Nuclear Physics, University of Madras, Madras-600025
"A Physicist's way of checking the corrctness of Mathematical equations"
17. D. (Mrs.) Bandana Ghosh, D. C. H. College, Dakhin Barsat, South 24 Parganas (W. B.)
"Unification of trilateral generating functions"
18. Mr. R. S. Chandel, Hoshangabad College, Hoshhangabad (M. P.)
"A common fixed point theorem of computing mappings in Saks space"
19. Mr. V. S. Dubey, Gorakhpur University, Gorakhpur,
"Decomposition of curvature tensor in R-recurrent finsler space with non-symmetric connection"
20. Dr Ashok Ganguly, G. S. I. T. S., Indore-452001, M. P.
"Banding Moments and Shear in Bernoulie"
21. Dr. C. B. Gupta, B. S. A. College, Mathura, U. P.
"Arithmatic and Geometry in Vedas"
22. Miss Archana Lala, Bundelkhand University, Jhansi, U. P.
"Inversion of an integral involving a generalised Hermite function"
23. Mr. Indrajeet Singh, A. P. S. University, Rewa, M. P.
"Some integrals involving the product of the double hypergeometric functions and multivariable H-function"
24 Dr. R.K. Prasad, Maharaja College, Arrah (Bihar)
"On approximate solutions of multivalued mappings in ordered Banach space"
25. Dr. S. K. Ghosh, Fellow Scientist. C. R. R. I., New Delhi
"On the mixed boundary value problem in the theory of a asymmetric Elasticity'.
26. Mr. Hemant Kumar, Telephone Exchange, Orai-285001,[U. P.
"A special class of polynomials."
27. Mr. Arvind Kumar Dixit, Govt. Higher Secondary School, Mobai, Tikamgarh, M. P.
'Vedic Mathematics"

28. Dr. H. C. Agrawal Bundelkhand College, Jhansi, U. P., "Some theorems on generating functions"
29. Dr. U. N. Singh, Magadh University, Bodh Gaya, Bihar "On approximate solutions of multivalued mappings in ordered Banach spaces"
30. Professor R. D. Agrawal, S. A. Technological Institute, Vidisha, M.P.
"An application of generalized Mehler Fock transformation"
31. Mr. Himanshu Pandey, Department of Mathematics, Univereity of Gorakhpur, Gorakhpur, U. P.
"Heterogeneous population growth model"
32 Dr Sambhu Nath Singh, Maharaja College, Arrah, Dist. Bhojpur, Bihar,
"On generating relations of polynomial set $\{U^*_n(x,y)\}$
33. Dr. Ramji Singh, Maharaja College, Arrah, Dist Bhojpur, Bihar
"Some integral represention of generalized polynomial set $\{M^*_n(x\ y)\}$
34. Dr B. N. Dwivedi, Atarra College, Atarra, Dist. Banda, U P.
"Hydro-magnetic stability of parallel shear flows between two parallel plates"
35. Dr. Hari Kishan, Atarra College, Atarra, Dist. Banda, U.P.
(a) Area of polygon
(b) Generalization of Feramat's last problem
36. Dr T. Dixit, Govt. Postgraduate College, Rishikesh,
"Accelaration covariance in M. H. D. terbulent dustry flow"
37 Dr. R Mall Govt. P. G. College, Rrshikesh,
'A note on MHD flow of an incompressible conducting fluid over an inclined non-conducting impervious boundary"
38 Dr. Kusum Shanker, Fellow Computer Centre, Central Building Research Institute, Roorkee-274667, U. P.
"A system Analysis approach to Town Planning:
A case study of National Capital Region, Delhi"
39. Rekha Mehndiratta, Cottage No. 4, Old Brockhurst, Shimla-9.
"Various fixed point theorems"
40. Ms. Pragati Goel, Research Scholar, Roorkee University, Roorkee, U.P.
"A two phase Mathematical model and Statical Analysis for drag reduction in laminar flow"

41. Dr. J. S. Verma Praveen Tech. Institute, Majra, Dehradun
"Geometry in Vedic Period"
42. Professor S. L. Singh, G. K. University, Hardwar, U. P., Local Secretary of the Conference
43. Professor S. C. Tyagi Principal, Science College, G. K. University Hardwar
"Welcome"
44. Dr. H. L. Gulati, G. K. University, Hardwar, U. P.
"On Job-block open shop scheduling problems"
45. Dr. P. P. Pathak, G. K. University, Hardwar, U. P.
"Decimal System in Time scale"
46. Mr. M. P. Singh c/o-Dr. S. L. Singh, G. K. University, Hardwar
"Vibration analysis of a non-uniform circular plate subjeced to one dimensional steady state temperature distribution
47. Mr. Vijayendra Kumar, G. K University, Hardwar
"२- दूरीक समष्टि में कुछ स्थिर बिंदु प्रयोग"
48. Mr. U. C. Gairola, G. K. University, Hardwar, U. P.
"A general coincidence theorem"
49. Mr. Ramesh Chand, G K. University, Hardwar, U. P.
"An extension of ancient Indian cube-root method"
50. Mr.Prem Lal Mishra, A. P. S. University, Rewa, M. P.
51. Dr. Srikant, D. R. D. O., New Delhi
52. Mr. Jai Narain Prasad, K. C. T. C. College, Raxaul-845305
53. Dr. S. K. Bhargava, 931-Civil Lines Jhansi, U. P.
54. Dr. J. S. Parihar, Atarra College, Atarra, Dist. Banda, U. P.
55. Dr. A. K. Agrawal, Sahu Jain College, Najibabad, U. P.
56. Dr. P. K. Mittal, Govt. P. G. College, Rishikesh
57. Miss Bharti c/o- Dr. Kusum Shanker, Fellow Computer Centre, Central Building, Research Institute, Roorkee-274667, U. P.
58. Miss Neeti Prakash, Meerut University, Meerut, U. P.
59. Miss Sarita Rani Sharma, G. K. Umiversity, Hardwar, U. P.
60. Mr. K. Manju Nath, Civil Engineering Department, Molrand Engineering College, Assan, Karnataka
61. Dr. Virendra Arora, Registrar, G. K. University, Hardwar
62 Dr. Rajendra Kumar, G. K. University, Hardwar. U. P.
63. Shri B. P. Shukla, G. K. University, Hardwar
64. Mr. Narendra Parashar, G K. University, Hardwar
65. Mr. Arun Kumar, G. K. University, Hardwar
66. Mr. Manoj Kumar Tyagi, G. K. University, Hardwar
67. Mr. Swami Nath Tiwwari, G. K. University, Hardwar

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MATHEMATICS IN VEDIC LITERATURE

By

SHASHI PRABHA ARYA
(Maitreyi College, Bapu Dham Complex, Chanakyapuri, New Delhi-110021)

First of all, I would like to thank the organisers of this Conference for inviting me here so that I could get this opportunity of talking to this august audiance.

In recent years, there has been a lot of talk about 'Vedic Mathematics' in the country at various levels. Several activities such as seminars, conferences, work-shops, round-table conferences etc. are taking place. But there seems to be a lot of confusion and controversy as to what really is meant by Vedic mathematics. You have already heard about what prof. Puri calls Vedic Mathematics.

What I am going to talk about is the real ancient vedic mathematics. This is *the* genuine vedic mathematics as contained in the Vedic literature. Again, there is a lot of disagreement as to which period is really to be regarded as the Vedic period. But a more accepted view seems to be that it is the period 2500 B C. to 500 B C. It is my intention in this talk to give you some glimpses of mathematics as encountered in the Vedic literature of this period. In the end, I would also like to make some remarks about the Vedic mathematics about which Prof. Puri talked and the controversey surrounding the same.

Like any other branch of Science, mathematics owes a great deal to the ancient Indian literature, the earliest available works being the Vedas. There is enough evidence that in the Vedic era, there existed a high order culture and civilization. Although sciences developed in

an attempt to help the religion, but soon they began to be studied for their own sake. Importance of mathematics was felt by all indian religions. In ancient India, the word *Ganita* used to be in vogue for the science of calculations. The following fomous verse of *Vedanga Jyotisha* indicates the special status mathematics enjoyed at that time :

" *Yatha Shikha Mayuranam, naganam manayo yatha Tadvat Vedangashastranam Ganita moordhanisthitan* "

That is, as the crests of the peacocks and the jewels of the serpents. mathematics stays at the head of all the *Vedanga shastra*. And ofcourse, who can refute the uses of mathematics. At no point of time, the civilization could do without mathematics. Not only religion, the day-to-day needs of a common man necessiated the study of mathematics in some form or the other. In *Chhandogya Upnishad*, there is a story in which Narad approaches sage Sanat Kumara and begs him to impart to him the *Brahm-Vidya*. Before doing so, the sage wanted to make sure that Narada deserved to know it. So he asked Narada to tell him what subjects he knew already. Narada then enumerated what he knew already and his list included *nakshatra vidya* (astronomy) and rasi-vidya (arithmetic). This clearly shows that *Ganita* was a subject of study during the vedic days.

Let me throw some light on what really is meant by the Vedic literature. The vedas are well-known as four in number : Rgveda, Samveda, Yajurveda and Atharvaveda. Each veda has four parts : *Samhitas*, *Brahmanas*, *Aranyaim* and *Upnishad*. As a necessary condition for mastering the Vedas, there arose six branches of study within the vedic schools : *Shiksha* (phonetics) *Kalpa* (rituals) *Vyakarana* (grammar) *Nirukta* (etymology), *Chhanda* (metric) and *Jyotisa* (astronomy). This group of literature is collectively called

Vedangas often referred to as the appendages of the vedas. *Vedangas* were written in the form of *sutras* (a phoristic style), a form of expression characterised by great precision, brevity and economy of words, the like of which is not met within the entire literature of the world . The style was developed to sum up only the pith of the learning in short sentences using nouns often compounded at great length and avoiding the use of verbs as far as possible . The style became a dominant feature of the various branches of *Vedangas* and was also adopted by the writers of *Arthashastra*, *Kamshastra* and *Natyashastra* .

The *Kalpasutras* are concerned primarily with the rituals which constitute the chief contents of the Brahmanas. These are supposed to be the first *Vedangas* to have received systematic treatment . The *Kalpasutras* are again available in four different classes : the *Srauta-sutras*. *Grhya-sutras*, the *Dharma-sutras*, and the *Sulba-sutras* . The *Srauta-sutras* deal with the sacrifices discussed at length in the *Brahmanas* . These are naturally concerned with direction for the laying of the sacrificial fires for the fire-sacrifice, the new and the full moon, the seasonal, the Soma and other sacrifices. These are our most important source for the understanding of the indian sacrifice cult . Through their preoccupation with the laying and construction of the various altars and fires, these also constitute an important source of Vedic, and possibly the earliest mathematics in India . Of special importance as far as mathematics is concerned, are the *Sulva-sutras* which are sometimes classified as a separate branch of the *Srauta-sutras*, but which are often found attached to the *Srauta-sutras* .

The *Sulva sutras* are of special importance because these deal specifically with rules for the measurements and constructions of

the various fires and altars and hence involve geometrical propositions and problems relating to rectilinear figures, their combinations and transformations, squaring the circle and circling the square, as well as arithmetical and algebraic solutions of problems encountered in connection with such constructions and measurements. The literal meaning of the word *Sulva* is to measure . Eventually, the word came to mean cord or rope. In ancient India, the word *Sulva* or *Rajju* (rope) was used for Geometry . The word Rekha-ganita came in usage much later .

At present, we know of only seven of the *Sulva-sutras* : *Baudhayana*, *Apaat hamba*, *Katyayana*, *Manava*, *Maitrayana*, *Varaha* and *Vadhula* . These were named after the *rishis* who had written them. All the Sulva-sutras exeept the Katyayana, belong to the section of Vedas called *Krishna-yajurveda*. The *Katyayana sutra* belongs to *Shukla yajurveda*. From the mathematical view-point, Bodhayana, Katyayana and Apasthamba are of significance . These were written between 800 BC and 500 BC. Other than these seven, there is no evidence of any other Sulva-sutras being in existence .

One is simply astonished to find that the geometrical-constructions as given in the Sulvas cover the first two books and the sixth book of Euclid. More so, since the Euclid's geometry is based on certain axioms and postulates and consists of results arrived at logically using the same . On the other hand, the logical methods of Greek geometry are not to be found anywhere in Hindu geometry . Besides, the sulvas are not formal mathematical works. They appear only as parts of ancient religious works .

Baudhayana's sutras are not only the earliest but represent the most systematic, logical and detailed treatment of the subject inspite

of their highly condensed sutra-style. *Apastamba* more or less provides the same rules and techniques. *Katyayana's* treatment is systematic. He emphasises the geometry behind the construction of altars and fires and gives a clear exposition of it. The *Manava-Sulbasutra* gives methods and details often very difficult to comprehend. In many cases, the details are either lacking or incomplete and can be understood only by reference to *Baudhayana, Apastamba* and *Katyayana*. On these four Sulva-sutras, the monograph by S. N. Sen and A K. Bag, published by the Indian National Science Academy in 1983 is particularly worth mentioning. They have presented the full critical sanskrit texts and the English translation of the four Sulva-sutras and modern commentaries on them. Another good reference is 'The Science of Sulba' by B. Datta published by the Calcutta University in 1932. Datta's article on vedic mathematics appeared 5 years later in 'The Cultural Heritage of India, Vol. III, Calcutta, 1937'.

There is enough evidence in the vedic literature that by 500 BC, the indians were well-versed in the use of numbers. They knew all the four fundamental operatioas of arithmetic i e. addition, subtraction, multiplication and division. Although these are not explicitly spelled out, but they were obviously taken for granted as common knowledge. For instance, in the *Rgveda* and the *Brahmanas* one comes across the case of dividing one thousand into three equal parts. In *Rgveda* it is not explicitly mentioned, but in *Taittiriya Samhita* it is clearly described as dividing the thousand into three. In *Satapatha Brahmana*, this operation of division performed by Indra and Vishnu is explained as follows: "For when Indra and Vishnu divided a thousand cows into three parts, there was one left, and here they caused to propagate herself in three kinds; and hence, even now if

anyone were to divide a thousand by three, one would remain over ."

The indians in the vedic era were deeply interested in thinking and naming large numbers . They adopted 10 as the basis of numeration . The various recensions of the *Yajurveda Samhita* give names to numbers as large as 10^{12}. The *Taittiriya Samhita* gaves the following numeral denominations :

Eka (1) , *dasa* (10), *sata* (10^2) , *sahastra* (10^3) , *ayuta* (10^4) , *niyuta* (10^5) , *prayut* (10^6), *arbuda* (10^7), *nyarbuda* (10^8), *samudra* (10^9), *madhya* (10^{10}) , *anta* (10^{11}) and *parardh* (10^{12}) . The same list occurs with some alterations in *Maitrayani* and *Kathaka Samhitas* and some other places . Infact, it seems that thinking out and naming large numbers was a favourite pastime of the ancient Indian mathematicians . The same tendency is noticed in the Buddhist literature where we find a centesimal scale (i. e. *Satotara Ganana*) and the name *talakshana* for the number 10^{53} the Jainawork *anuyogadwara sutra* (100 BC) called the places in powers of 10 as *gananasthan* and mentioned large numbers upto 29 places and beyond . The jains are further credited with the conception of a time-scale called *sheersh-prahelika* (i, e. 84 hundred thousand) and the suggestion of building up fantastically large numbers in aseending powers of this figure . But let us come back to the vedic period .

The Indians in the vedic period developed a scientific vocabulary of number names using the principles of addition, subtraction and multiplicntion . The system required the naming of (i) the first group of numbers i. e. the first nine digits i. e. *eka*, *dwi*, *tri*, *catur*, *panca*, *sat*, *sapta*, *asta* and *nava*, (ii) the second group of nine nambera obtained by multiplying each of the above digits by 10, i. e. *dasa*,

vimsati, trimsat, catvarimsat, pancasat, saptati, *asiti , navati .* The multiplicative principle is already indicated in forming the numbers of this group . In namig a number in which both these groups participate, additive, or additive and multiplicative principles both, are simultaneously used, for instance, *eka-dasa* (11) or sapta-vimsati (720) .

The numbers were classified into two classes : yugma (even) and ayugma (odd). Also, in the Atharvaveda, there seems to be a reference to zero as also to the recognition of negative numbers .

Infact Brahmagupta, a later Hindu mathematician is credited with the use of zero in its different mathematical operations, but a text namely *pingala chhanda sutra* is said to mention a symbol for zero . Sometimes different words were used for expressing the same numeral . For instance, zero was expressed by the word *shoonya or gagan, ambar, akash,* 1 was expressed by chandra, soma, prithvi or pitamaha, likewise 2 by *ashwina*; *netra* or *chakshu* etc, 3 by *triguna, loka, trijagat* etc . 4 by *sruti, varga, yuga,* etc., 5 by *tatwa* or *erana,* 6 by *anga, ritu* or *darshan shastra* and 7 by *rishi, muni* or *swara .* Symbol-system for writing numerals was also used. In rgveda, thare is a verse which has been interpreted to mean "Give me a thousand cows on whose ears the number 8 is written." There must be some symbol for the number 'eight' which was marked on the ears of the cows-whichwas a practice followed in later times also. In another place in rgveda, a gambler says "Having staked on 'one' he lost his wife", which also indicates that there was a written symbol to denote the numer one. (Regarding measures and weights, vedic measures were, simple, meeting the needs of day-to-day life . The term *pramana* meaning measure was classified into four types, namely *mana* (measure

of weight), *avamana* (linear measurement) and *kal pramana* (measurement of time). For linear measurements, different parts of human body were regarded as units, A man's stride was known as *prakrama*. Similarly, measure of capacity *prasrishti* meaning a handful was used. Similarly *anjali* meaning two hands joined together which was double of *prasrishti* was also in common use . Sulva-sutras refer to a variety of linear and area measurements .

The vedic literature clearly indicates acquaintance with the fundamental arithmetical operations with elementary fractions In the rgveda one finds names of a number of simple fractions such as *ardha* (1/2), *tripada* (3/4) and the *maitrayani samhita* mentions *pada* (1/4), *sapha* (1/8) *kustha* (1/12), *kala* (1/16) . From the sulba-sutras onwards, we meet with terms such as *amsa*, *bhaga* to denote fractions in general . These terms are used in combination with cardinal or ordinal number names. In the case of ordinal number names, the word *amsa* or *bhaga* is often omitted. For example,

tribhaga or *tryamsa* for 1/3
pancama-bhaga or *pancama* for 1/5
dvadasa-bhaga or *dvadasa* for 1/12
pancadasa-bhaga for 1/15
tri-astama or *tryasta* for 3/8
dvi-saptama for 2/7
trayastryasta for $3\frac{3}{8}$
pancamasya caturvimsa for 1/24 of 1/5, etc.

The *Sulva-sutras* contain several examples of addition, subtraction, multiplication, division and squaring of fractions . For instance in

Baudhayana one comes across the following passage :

"Then he measures the area of this square shaped *siti*. whose side is three *purusas* less one third. At the western side of this square is the handle whose east-west length is half a *purusa* plus ten *anguls* (i. e. one-twelfth *purusa*) and north-south breadth one *purusa* less onethird. This makes the *agni* sevenfold plus two *aratnis* and the *pradesa*". Here two *aratnis* and the *pradesa* equal 60 *angulas* or 1/2 *purusa*. Expressed in figures, this means :

$$\text{The area of the } agni = (3 - 1/3)^2 + (1/2 + 1/12)(1 - 1/3)$$
$$= 7\tfrac{1}{2}.$$

Several such statements can be traced in the *Sulva-sutras*.

There is evidence in *samhitas* that the indians in the vedic period were interested in series or progressions of numbers also. In *taittiriya samhita*, one finds the following series :

1, 3, 5, ... 19, 29, .., 99
2, 4, 6, ..., 20
4, 8, 12, ..., 20
5, 10, 15, ..., 100
10, 20, 30, ..., 100 etc.

The arithmetic series were classified into *yugma* (even) and ayugma (odd) series. The following examples of these two categories are given in the *vajasaneyi samhita* :

4, 8, 12, .., 48
1, 3, 5, ..., 33.

Of these two series, the second one is found to occur also in *taittiriya samhita*. In the *pancavimsa brahmana*, there is given a list of sacrificial gifts which form the following series in geometrical progression :

24, 48, 96, 192 , ... , 49152, 98304, 196608. 393216 . This series also makes its appearance in *Srauta-sutras*. In *satapatha Brahmana* summation of such series is also given. For instance, the sum of the series .

$3x$ (24 + 28 + 32 + ... + to seven terms) is correctly indicated as 756 . In the *Brahaddevata*, the sum of the series

2 + 3 + 4 + ... + 10000 is given correctly as 500499. Although no rule for summing such series is mentioned there, but one is led to believe that the indians in the vedic era must have known some method for summation of both, the arithmetical and the geometric series .

There are several results in the Sulva-sutras which suggest that the indians of the vedic-time knew how to perform fundamental operations with fractional numbers .

A study of vedic-literature reveals that so far as the geometry is concerned, the indians at that time bad obtained several results concerning rectilinear figures, the mensuration of areas and volumes of these figures etc . They had a special fascination for the pythagoras theorem, which in *Baudhayana* is stated as follows :

"The diagonal of a rectangle produces both (areas) which its length and breadth produce separately." That is, the square described on the diagonal of a rectangle has an area equal to the sum of the areas of the squares described on its two sides . The converse of the pythagoras theorem, although not mentioned explicitly, was cleverly used throughout in the construction problems of fire-altars . Many a times, figures had to be first converted into squares and then the Pythagoras theorem and its converse were repeatedly used to arrive at the desired result .

Elementary treatment of surds is found at various places in the

vedic literature. $\sqrt{2}$ has been referred to as *dvi-karani*, $\sqrt{3}$ as *tr-karant*, $\sqrt{4}$ as *catuskarani* etc. The irrationality of $\sqrt{2}$ was recognized and its approximate value as calculated then woks out to be 1.4142156 which is correct upto fifth place of decimals. It is amazing how the value of $\sqrt{2}$ was obtained to such a degree of accuracy in those early days.

In an attempt to determine the size and number of bricks needed for building the different layers of the altars. simultaneous indeterminate eqnations of the first degree were encountered. This necessiated the methods of solving quadratic equations and indeterminate equations etc. In ihe vedic-literature, one finds references of permutations and combinations. There were problems such as the total number of perfumes that can be made from sixteen different substances taken one, two, three or four substances at a time and so on.

In the closing phase of the vedic,period the Jainas deserve special notice for their interest in mathematics, Their literature makes refernce to various mathematical topics such as *sankhyayana* (Science of numbers), *parikarma* (fundamental operations), *vyavahara* (subjects of treatment), *rajju* (gaometry) *rasi* (heap, solid mensuration), *classvarna* (fractions), *yavata-tavata* (equations, algebra), *varga* (square, quadratic equations), *ghaaa* (cubc cubic equations), *varga-vnrga* (biquadtatic equations) and *vikalpa* (permutations and combinations).

Now, I would like to make a few comments on the book entitled Vedic Mathematics or sixteen simple mathematical formulae from the vedas written by Jagatguru Shri Bharti Krishna Tirathji Maharaja, Shankaracharya of Goverdhan math of Puri about which a lot has been said already. The book wrs first published by the Banaras Hindu

University in 1965 as volume 10 of Hindu Vishwavidyalaya Nepal Rajya Sanskrit series. Subsequently, the book has been reprinted ten times, the latest edition appearing in 1988. Because of the word vedic mathematics in its title one gets the impression as if the mathematics contained in the book is the mathematics of the vedic era. However, this is not true. The results of the book are to be found no where in the vedic literature. V. S Agrawala, the general editor of Swamiji's book, was also aware of this fact. But because of his regards to Swamiji he wrote as follows :

"The question naturally arises as to whether the sutras which form the basis of this treatise exist anywhere in the vedic literature as known to us. But this criticism loses all its force if we inform ourselves of the definition of veda given by Swamiji himself as quoted below :

'The very word veda' has this derivational meaning i. e. the fountain-head and illimitable store-house of all knowledge. This derivation, in effect, means, connotes and implies that the vedas *should* contain within themselves all the knowledge needed by mankind relating not only to the so called 'spiritual' or other worldly matters but also to those, usually described as purely purely 'secular', 'temporal' or 'worldly' and also to the means required by humanity as such for the achievement of all-round, complete and perfect success in all conceivable directions, and that there can be no adjectival or restrictive epithct calculated (or tending) to limit down that knowledge in any sphere, any direction or any respect whatsoever. In other words, it connotes and implies that our ancient Indian Vedic lore should be all-round, complete and perfect ard able to throw the fullest necessary light on all matters which any aspiring seeker after

knowledge can possibly seek to be enlightened on". It is the whole essence of his assessment of vedic tradition that it is not to be approached from a factual stand point, but from an indeal stand point i. e. as the vedas as traditionally accepted in India as the repository of all knowledge *should be* and not what they *are* in human possession. That approach entirely turns the tables on all critics, for the authorship of vedic mathematics then need not be laboriously searched in the texts as preserved from antiquity".

In his preface to his book, Swamiji has stated that the sixteen Sutras dealt with by him in that book were contained in the *Parishishta* of the Atharvaveda. But this is also not a fact, for they are not to be found anywhere in the known *parishishtas* of the *Atharvaveda* edited by G. M. Bolling and J Von Negelein. According to Professor K. S. Shukla, sometime in 1950's when Swamji visited Lucknow to give a black-board demonstration of the sixteen Sutras of his book at the Lucknow University. Prof. Shukla personally wenr to him at his place of stay with Bolling and Negelein's edition of the *Parishishtas* of the *Atharvaveda* and rəquested him to pinpoint the places where the sixteen Sutras demonstrated by him occured in those *parishishtas* Professor Shukla narrates that Swamiji replied offhand, without even touching the book that the sixteen Sutras demonsitrated by him were not in those *parishishtas*, they occured in his own *parishishta* and not in any other.

V. S. Agrawala has also given the verdict that the work of Swamiji deserves to be regarded as a new Parishishta by itself. However, the question is whether any book written in modern times on a meodern subject can be regarded as a Parishishta of a Veda. The answer is definitely in the negative. It may be '*good* mathematics' but certainly not '*vedic* mathematics'.

According to this view, it seems that the sixteen Sutras of Swamiji's 'Vedic mathematics' are Swamiji's own creation and have nothing to do with the mathematics of the vedic period. Infact, Swamiji goes a little too far in his claim. He designates his preface to the book as 'A descriptive prefactory note on the astounding wonders of Ancient Indian Mathematics'. Not only that, at several places he calls his mathematical processes as vedic processes. Now the point is that a general reader has no access to the vedic literature. It is therefore not possible for him to actually verify Swamijl's claim that the *Sutras* as given in the book appear somewhere in the vedas. No wonder then that there is a general impression (unfortunately even at a high level) that the mathematics as contained in the vedas consists of the sixteen *sutras* of Swamiji's book.

Professor K.S. Shukla comes out in a big way refuting Swamiji's claim. He has quoted several instances from the book supportiug his views, some of which are as follows :

In chapter one, conversion of vulgar fractions into recurring decimal fraction is given Nobody in the vedic period could think of a recurring decimal. The methods of multiplication and division are quite different from the traditional Hindn methods. The theorems of Leibnitz, Maclaurin aad Taylor, integration by partial fractions dealt with in the book are all modern topics. The system of expressing numbers by means of letters of the Sanskrit alphabet has been called by Swamiji 'the vedic numerical code' although it has not been used anywhere in the vedic literature.

Dr. Puri remarked that how can we say that these Sutras are not contained in the several lost unpublished *pandulipies*. But as I understand, vedas are our sacred books which have not been allowed

to be tempered with. Either the Sutras are there and if they are not there, they cannot be called vedic. Mathematicians do not make statements in the air. Nor do they believe such statements. For everything, there must be supporting evidence-either a proof or a counter-example. As Professor Kapur also said and as you must also have felt in view of what I have said that here is a vast gap in the type of mathematics contained in vedas and the vedic mathematics of ancient time. Also there is no circumstantial evidence available in support of Swamiji's claim. One should be able to show that if not explicitly, atleast implicitly the Sutras are there in the vedas. Unless that has been done, I don't think there is enough justification for calling it vedic mathematics.

Inspite of all this, one must accept that the *Sutras* given in the Swamiji's book are beautiful, They do make calculations amazingly simple and fast in some particular. The book has inspired work in some new directions The late Professor P. L. Bhatnagar had done an indepth study of Swamiji's book and published a long paper in the Mathematics Teacher. In this paper he tried to *prove* some of the *Sutras*. Some more work has to be done in that direction, There is a suggestion of introducing *Sutras* as part of regular school curriculum. But here again we have to be careful. Unless we can show the universal applicability of the results, they cannot be introduced as part of the syllabus. As enrichment material. one con recommend Swamiji's book without any hesitation. Efforts are afoot to establish and investigate the origin of the sixteen Surras.. At a recent round table conference in Delhi organised by the *Bashtriya Veda Vidya Pratishthan*, there was a suggestion to study these sutras as vedic mathematics of J. S. The *Pratishthan* had earlier organised two national workshops-one at Jaipur and the other at Ahmedabad. As

regards the literature, the National Commission for the compilation of the History of Sciences in India is doing commendable work. Due to its efforts, some good books have been published. For example, 'A concise history of Science in India' edited by D.M Bose, S.N *S*en and B.V *S*ubharayappa published by the Indian National Science Academy in 1971 is an excellent sourcematerial. It gives a concise account of the development of *S*cience in India from ancient times to the 18th century. There are 11 chaptars on different subjects in the book, mathematics being one of these. Another book in the series on sulba-sutras by *S*en and Bag published by INSA in 1983 I metioned earlier.

For the last 30 years, A. Seidenberg has been making serious study of vedic mathematics at part of his general theory of ritual origin of mathematics. His 42 page paper on 'The origin of Mathematics' (Arch. Hist. Fxact *S*ciences, 1978) includes a speaial Appendix on 'Vedic Mathematics',

To sum up, the *genuine* vedic mathematics is that mathematics which is contained in the vedic literature in the period 2500 BC to 500 BC. The indians in the vedic period were well-versed in the use of numbers. Large numbers sueh as 10^{12} were known to them They had developed a scientific vacabulary for number names. Symbols were also used for writing numerals, They knew all the four fundamental operations of Arithmetic. They were familiar with simple fractions. They were interested in series of numbers. In geometry, they solved propositions about the construction of various rectilinear figures, mensuration of areas and volumes etc. No doubt, this vedic mathematics is of great historical significance for studying and understanding the science and culture of India about 2000 to 5000 years ago. The modern system of vedic mathematics as

propounded by Jagadguru Shankaracharya, howsoever beautiful it might seem to be, should not be confused with the real ancient vedic mathematics a glimpse of which I have tried to give, One must also be cautioned against the exaggerated extreme belief that all modern discoveries and inventions are hidden secretely in the vedas. One must more with care, Let us understand clearly: Vedic mathematics is only that mathematics which is contained in our sacred vedic literature.

VEDIC MATHEMATICS – MATHEMATICS OF SMILES

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INTRODUCTION

I wish to draw your kind attention to another wonderful mathematical facet of rich, vast and scientfic cultural heritage of India. Vedic Mathematics, derived from the Veda, provides one line, mental and superfast methods alongwith quick cross checking system. As such Vedic Maths is a boon for all competitions. Vedic Maths converts tedious Maths into a playful and bliss ful subject, which students learn with smiles.

SALIENT FEATURES

Vedic Maths (VM offers a new and entirely different approach to mathematics based on pattern recognition for resolving the current crisis in Maths education allover the world. VM allows for constant expression of a student's creativity and has been found to be easier to learn For any problem there is always one general technique applicable to all cases, There are also a number of special techniques for special pattern problems. The element of choice and flexibility at each stage keeps the mind lively and alert and develops clarity of mind and intuition, and thereby Holistic development of the hnman brain automatically takes place.

UNIQUE FEATURES-IN BRIEF

1. BINDU MEIN SINDHU

Vedic Approach of Bindu Mein Sindhu : In Vedic thought we

always think of large number of possiblities and we consider that each small drop essentially represents an ocean–Bindu Mein Sindhu. Further, both negative and positive coexist to make the play go on. Perfectly, in tune with this, we find that in Vedic Mathematics, we have both positive and negative digits and a number consisting of both positive and negative digits is called vinculum number .

As we learn the numbers we also learn their complements . This then immediately provides a possibility of seeing any digit in two alternative forms. For exomple

$$8 = 10 - 2 = 1\bar{2} .$$

The question naturally comes up if there is some other possibility to express 8 which still the value of 8 .

VM provides yet another possibility as

$$1 = 1\bar{2} = 1\,\bar{9}\,\bar{2}\ (100 - 92)$$

LINK WITH ALGEBRA : Once we become very familiar with the vinculum numbers and their operation, then we can see a perfectly convenient link between Arithmetic and Algebra . A polynomial

$1\,x^3 - 9x - 2$ is just one algebraic representation of number 8 . (with $x = 10$, as we are working in the decimal system) . as $1\,x^2 - 9x - 2 = 1\,\bar{9}\,\bar{2}$

Therefore, in Vedic system, all operational procedures of Arithmetic are straight away used in Algebra which provides tremendous amount of simplicity, ease and convenience to the student . Further, Vedic Mathematics provides the clear illustration of the Vedic concept, 'Bindu Mein Sindhu' .

2. MATHEMATICS – A VITAL SUBJECT

Whatever we want to do in life we always have a large number of

alternatives and choices available to us. This element of choice makes the mind lively and alert , However if we are asked to do the same thing in a fixed order following a rigid sequence of steps, even a very convenient and interesting task shall soon become boring and tedious and is very likely to result in unnecessary anxiety and boredom. On the other hand, the availability of several alternatives and choices gives the opportunity of freedom of selection and makes the whole task interesting. This element of choice makes the mind alert and agile, as the mind tries to follow the path of least resistance and gives the individual opportunity of self expression in varied ways .

For simple arithmetical operations we have a unique fixed and general method of addition, subtraction, multiplication, and division. Same is true for most of the higher aspects of Mathematics. We spend well over 5 years mastering these four basic arithmetical operations. This fixity and rigidity with little possibility of any type of annovation, freedom and choice leads to monotony which in turn creates the unnecessary problem of MATHS ANXIETY. A common student is afraid of Maths . He is afraid of the most important, fundamental and vital subject - MATHS.

Children love to play games . As a matter of fact we all like one or the other game . If we could teach the child one method of multiplication in class 2nd. Give him a second method of multiplication in class 3rd; add another alternative after six months for very conveniently multiplying the numbers closer to the base; teach him multiplication using Vinculum numbers (consisting of both positive and negative digits) in class 4th; which would require least computational effort; and after he has practised well, multiplying from right to left, the student learns the procedure of multiplying from left to right . With the result that even process of multiplication

becomes a game for a poung child. This exactly has been the response of the students of the schools of England, Holland and America, where Vedic Mathematics (VM) is being taught .

Dr. Robin Rowe Academic Director of group of MSAE schools at Fairfield in Iowa state (U. S. A.) reports; "The children are bubbling with *joy and bliss* in Vedic Maths class" .

So we find that Vedic Mathematics with its special features has the inbuilt potential to solve the problem of Maths Anxiety .

3. MOST EFFICIENT COMPUTATIONS

All of us have this experience, that whenever we are operating the bigger digits (6, 7, 8. 9), it requires slightly more effort and a little more time, compared with the operation of smaller digits . This difference may be only in microseconds , but definitely it is slightly more straining to operate bigger digits ,

The recent medical research on human brain has lead to the conclusion that the information in human brain is stored in memory registers . The front most register can store only five bits of information. Therefore, when we are using the first five digits (1 to 5) only the front most register is used, hence the ease and convenience, but for operating the bigger digts (6 to 9) the second level register of human memory has to be used. As such it requires a slight more effort and time .

The whole Vedic tradition is based minimization of efforts on the part of the human being, which is the roof and crown of God's creation . Further. in VM it is possible to do the entire computations by using only the front most register waich can store five bits of information .

In Vedic Maths we have a choice to use the digit 8 as it is or alternatively use $\overline{1}2$. Similar is the case with other digits. These are referred to as Vinculum numbers.

Therefore, in VM it is possible to carry out the entire computation process using only the front most memory register by operating the smaller digits. We shall all agree that operating 1 and $\overline{2}$ is far more easier, simpler and quicker than handling 8 Further, the chances of error are drastically reduced. As a matter of fact when we are operating only smaller digits we also have an opportunity to smile.

4. MULTIPLE CHOICE

In the present system of Mathematics we have most of the time, a unique way of solving the problem. This has the obvious advantage of convenience of learning the procedure in a shorter time, but this unique fixed way of working very often leads to monotony. There seems to be no possibility of improvement of computational efficiency in any way. Vedic Maths on the other hand provides multiple ways of solving any problem. Once we learn the three Vedic ways of multiplying numbers. The mind naturally thinks there should be some other way also and soon we find that we can multiply in six different ways. We have a choice to work from left to right or right to left. Further, we can convert the bigger digits into Vinculum form.

5. CHECKING METHOD

One can easily check the correctness of the result obtained by redoing the problem using another way of solving.

Alongwith the multiple choice of working, VM also provides us with independent cross checking methods. These checking methods provide a convenient opportunity to smile at the end of each problem.

6. COMBINED OPERATIONS OF ARITHMETIC

VM emphasizes the mental working and provides an opportunity to carry out the combined operations of Arithmetic in a single step. With practice the extent of combinations keep on growing. Sum and difference of the products of a few pairs of numbers can be done directly in one line etc.

The possibility to carry out the combined operations of arithmetic has tremendous applications in different areas of Modern Algebra and Higher Mathematics .

7. OBTAINING DIRECT SOLUTION

Presently, in many operations like multiplication, squaring, cubing, adding etc., we work form right ro left. The left most digit is almost always the most significant digit and the second digit from left is the next significant digit . Since we have to work from right to left, it is almost necessary to carry out the entire computations for getting the most significant part of the answer . In Vedic system we have a complete choice to start our computations either from left or from right (AVM Pushp 2 and 3) . It just does not matter whether we are multiplying or squaring or solving a transcendental equation. The capability to start our working from the left can be very usefully utilized in handling problems of diversified areas of Mathematics . Once we start our computations from the left, we are in a position to obtain the most significant digit in the very first instant . Then we use just the digits already obtained for computing the second most significant digit in the very first instant . Then we use just the digits already obtained for computing the second most significant , In this way we can go on with our working till we obtain the desired level of accuracy and precision .

8. HIGHER APPLICATIONS

Vedic Mathematics system provides an integrated approach to different areas of Mathematics As the same sutras are applied directly for diversified areas like Trigonometry-Plain and Spherical, Positional Astronomy, Coordinate Geometry 2D and 3D, Differential and Integral Calculus etc. We only need to add very few additional concepts and formulae. The recent studies indicate that most of the problems in these areas can be directly solved and it appears that bulk of the formulae and identities which we use at present become redundent (Refer Higher Applications of VM).

9. INDEPENDENT CHECK ON COMPUTER RESULTS :

Vedic Mathematics is the only convenient and useable system which can provide an opportunity of independently checking the computer results. Further, in several special pattern problems the results are obtained with a dramatic speed due to the simple and convenient Vedic procedures. Therefore, Vedic Mathematics system really appears to be the system compatible with the present day computer age.

10. DEVELOPMENT OF INTEGRATED PERSONALITY

According to the brain specialists, the left half of the brain. systematically collects the information, does sequential analysis and arrives at the logical conclusions and results. The right half of the human brain works with pattern recognition and has the capability of intuition. This most powerful facet of human personality the intuitive faculty, unfortunately, remains undeveloped in most of us. Can right brain thinking be taught ? Yes, say the experts but it takes

time and practice. With the left brain silenced the holistic right moves into action.

The Vedic Mathematics system provides us with a large number of options at each stage of working. Depending upon the pattern of the problem, one of these options is going to give the result with very little effort and in minimum time. Therefore, the very first step is to look for the pattern of the problem, which essentially involves systematic use of the right half of the brain. Then the logical computations are done by the left half. Even while computing, due to various options available, the mind is always kept alert to pick up the path of least effort. This in turn brings joy in the heart and smile on the face of the students. Therefore while practising Vedic method human brain gets the systematic training of integrated functioning of the two halves of the brain. This according to the educational experts is very likely to result in Holistic Development of Human Personality. In this context, the feed back from the American schools, where systematic teachings of Vedic Mathematics has already been introduced is thought provoking.

The head of Maths department from St. James School London (where they are systematically and *exclusively teaching Vedic Mathematics since last 8 years*) has informed that, ' The general performance of our students in Maths is excellent and it is helping them in development of their personality. Because the Vedic Methods take proper account of how the hnman mind works the calculation is done faster and leaves the mind bright and full of energy. All of us find that the effect on the boys is to make them very bright.

One boy shouted back an answer so quickly that the teacher asked, "How did you do it ?" He said "It just flashed in my awareness" This is the working of the intuition faculty.

In a nutshell we can say that the Vedic Maths provides the *cosmic software* for the *cosmic computer* (*the human brain*) creating coherence in the brain physiology, so that the later is not wasted ,

CURRENT DEVELOPMENTS IN INDIA

The interest in Vedic Mathematics, as reexpounded by Shankra Charya Bharti Krsna Tirth ji, has been steadily growing in the last few years . The book by Shankracharya was originally published in English by the Banaras Hindu University . Its translations in Marathi and Gujarati have already been published . Further, 20 books have been published in India and 5 books in U. K. ,

(i) Ministry of Human Resource Development (MHRD) has started the Rashtriya Ved Vidya Pratishthan (RVVP) which has already organised two national workshops, and a Round Table discussion on Vedic Mathematics . RVVP has set up a national committee to plan further Vedic Maths activities .

(ii) The National Council of Educational Research and Training (NCERT) has identified Vedic Mathematics as one of the thrust areas and included Vedic Mathematics ia the teachers training course and teachers manuals for enrichment material in Maths education .

(iii) The Vedic Maths Research Group (VMRG) was formed at the Roorkee University in 1988 and is organising courses and research in Vedic Mathematics . The Spiritual Study Group (SSG) of Roorkee has been active in organising VM lectures, short courses and correspondence courses in different parts of the world . SSG has published series of books and released video and audio cassettes .

(iv) A series of training courses and workshops are being organised in Madhya Pradesh. Maharashtra, Gujrat, Rajasthan and

Uttarpradesh. Series of Vedic Maths programmes are being shown under UGC National class room, T. V. programmes .

(v) Systematic teaching of Vedic Mathematics has been started at a few schools . The secondary education department of Madhya pradesh has permitted teaching of Vedic Mathematics to class IX and X at the Government schools at Mawai, Tikamgarh on experimental basis .

VEDIC MATHEMATICS RESEARCH GROUP ROORKEE UNIVERSITY

The Vedic Maths Research Group (VMRG) at Roorkee university was formely established in october 1988 with Prof . Anand Swarup Arya, Pro-Vice Chancellor, Roorkee University as the Chairman . Prof . S. N. Pandey, Head of the Maths department and Prof . G. C. Nayak, Head of the Civil Engineering Department and coordinator CAD-CAM centre of Roorkee University as the Co-chairman. Dr. N. Puri. Reader in Civil Engineering Department is the Secretary . Further, VMRG has members from the departments from Civil Engineering, Computer and Electronics Engineering, Mathematics, Mechanical Engineering, Metallurgical Engineering, Hydrology and Architecture .

The Spiritual Study Group (SSG), consisting of the students and staff of the Roorkee University and now having several outside members, started the study of Vedic Mathematics in March 1985 . The response of the participants was excellent . This resulted in a series of Vedic Maths courses .

SSG has further started the Instiute of Vedic sciences (IVS) . IVS has been conducting short courses and correspondence courses on

Vedic Mathematics. *SSG* has been publishing a large number of books and workshop notes on Vedic Mathematics. Further, a series of Video and Audio cassettes on lectures and courses of Vedic Mathematics have also been released.

The VMRG has been conducting the Vedic Maths courses for students and teachers. The members of VMRG prepared a set of 8 research papers on Higher Applications of Vedic Mathematics for the Ahmedabad workshop of **R**VVP. VMRG has prepared and submitted major research projects on, Vedic Mathematics and its applications in Civil Engineering and Computer Graphics, for the University Grants commission and Department of Science and Technology of the Government of India.

VMRG is also preparing special books for Engineers, teachers, bankers etc.

The MHRD established the Rashtriya Ved Vidya Pratisthan (RVVP) in 1987. MHRD, RVVP, University of Rajasthan alongwith the Pondichery University organised the first national workshop on Vedic Mathematics at Jaipur in March 88. The Workshop accepted Vedic Mathematics as a very interesting and useful subject and recommended the setting up of a National committee. It felt the need to provide funds for further research and development of the subject and spread the available information.

This was followed by the second workshop held at Ahmedabad in Oct. 88. 8 Theme papers on applications of Vedic Mathematics in Algebra, Trigonometry, Coordinate geometry, Calculus and solution of linear and non linear integro-differential equations were prepared by the Vedic Maths research group of Roorkee University for the workshop, The question of introducing Vedic Mathematics at the

primary level was also discussed alongwith highei applications of VM. The simplicity and case provided by Vedic Maths in trigonometry, both plane and spherical, was highly appreciated. The MHRD further held the Round Table discussions (RTD) on Vedic Mathematics in Feb . 89 at Delhi . The RTD gave the following recommendations .

The Round Table Discussion organised by the Rashtriya Ved Vidya Pratishthan from 22nd February to 24th February, 1989, has come to the conclusion that the Sutras in the book 'VEDIC MATHEMATICS' of Jagat Guru Shankaracharya Shri Bharti Krsna Tirtha ji, are interesting and useful . While further theoretical and experimental investigations are necessary before a decision can be taken for introduction in the curriculum, the following positive steps are recommended :

(i) The national council of educational research and training may be requested to include some of the sutras and their adplications as enrichment materials, in consultation with a panel of mathematicians constituted for the purpose by the Rashtriya Ved Vidya Pratishthan.

(ii) Agencies inclined to conduct training programmes in Vedic Mathematics should be encouraged and supported .

(iii) The Rashtriya Ved Vidya Pratishthan should commission the writing of the following-

(a) A book giving for each sutra, its statement, its proof, its motivation, illustrative examples, explaining limitations and possibilities for generalisation .

(b) A book explaining how students can discover the sutras by recognising patterns with the help of illustrations .

(c) A book explaining sutras for educating teachers and other

users like managers, bankers, engineers, etc .

(d) A book comparing the Sutras with other methods of calculations and in particular. with those of trachenburg speed system of Basic Mathematics .

The above books should be Wetted by a panel of mathematicians before permitting their publication .

(iv) Projects may be sanctioned to competent mathematicians and computer scientists .

(a) Development of ideas in the book 'Vedic Mathematics' by Jagat Guru Shankaracharya .

(b) Investigating the possibilities of incorporating the relevant ideas in appropriate computer software programmes .

(c) Expounding and developing Mathematical Foundations behind the sutras ,

CURRENT DEVELOPMENTS ABROAD

Vedic Maths work was started in England in 1980. The Vedic Maths research Group at London has been organising lectures/courses and published a set of 5 books . The St. James school London has been systematically and exclusively teaching Vedic Mathematics from Class-I to VII for well over 5 years . Two more schools ; one in Dublin and the other in Amsterdam have also started the teaching of Vedic Maths .

Dr. N. Puri went on a comprehensive 18 nation tour of Vedic Mathematics in early 1988 . This was followed by two regional VM training courses in June and July, 1988 at Switzerland and America . As a follow up of this a number of Vedic Maths lectures/courses are

being organised in different parts of the world. Further, systematic teaching of Vedic Maths have been started in a number of schools at IOWA, Washington D. C., Mexico etc. Excellent response is being received from all places.

VEDIC MATHEMATCIS RESEARCH GROUP AT U. K.

The VMRG at UK has been active since 1980.

Courses held

University of London Union, 1980-1983.

Polytechnic of North London : Easter 1982 : 1983 / 84 and 1984 / 85.

Mary Ward Centre : 1985 / 86 and 1986 / 87.

Working Men's College : 1985-86.

First Vedic Mathematics visit to India (1981 by A. P. Nicholas.

Tour of India : Visiting Universities etc. seeking information, giving lectures

From : 22nd July 1981 To : 23rd Sept. 1981.

Books : Five books have been brought out from U. K.

Tour of U. S. A.

From : 7th Sept. 1983 To : 21st December 1983

Lecture / Workshops were given at the 18 institutions / universities

Vedic Mathematics Lecture Tours of Holland (17th October - 4th November, 1988) and West Germany (7-11th November and 6-24th March 1989).

Research Paper :

Pickles, J. H. Nicholas, A. P. and Williams, K. R., (1982) "Bharati Krsna's Systems of Vedic Mathematics", The Institute of Mathematics and its Applications, Vol. 18, 233-236.

INTERNATIONAL CORRESPONDENCE COURSE ON VEDIC MATHEMATICS

The correspondence course on Vedic Mathematics is being run by the SSG, Roorkee since Haridwar Mahakumbh 1986 in English and Hindi. The course was of six months duration and consisted of two parts. From 1990, the course shall be of 1 year and shall have 3 parts . There are 6 fortnightly lessons in each part. New batches start from auspicious Baishakhi (April) and Dashahara (October)every year.

The first part of the course covers basic arithmatic . The second part covers basic Maths and Algebra. The third part covers the Vedic methods for trigonometry (Plane and Spherical), Coordinate Geometry (2D and 3D), Determinants, Calculus and solution of linear, nonlinear differential and integro-differential equations, It also includes the scheme for systematically teaching Maths in the vedic way ,

A number of participants from different countries have also been joining the correspondence course .

VEDIC MATHEMATICS WORLD ACROSS

Dr, Narinder Puri went on a 7-week world tour in March and April, 1988. He visited 18 countries of Europe and Ameriea including England (London, Oxford, Cambridge, Lancashire), USA (Washington, Boston-Harvard Univ., Chicago, Los Angeles, Stanford University, PolaAlto, Berkeley, San Francisco, Indiana, Fairfield, MIU), Canada (Toronto University) Mexico, Norway (Oslo and Lily Hamer, Sweden (Stockholm and Upsala University), Finland (Turku University), Denmark (Copenhagen University and H. C. Oersted Institute), Germany (Hemburg University, Berlin . Free Univ and Technical Univ.), Hungary (Budapest, Royal Institute of Leadership). Austria (Vienna University), Italy (Rome, Naples University, Milan

University and Como), Switzerland (Seelisberg MERU, Geneva University), France (Institute Henri Poincane-Paris), Spain (Barcelona University), Holland (MERU) and Belzium. Dr. Puri delivered 101 hours lectures during this tour. Dr. Puri was interviewed by several TV, Radio and Press media during his tour. After magnificant developments in Hardware and Software technology, there is an urge for Brain ware development. It requires a totality of brain, where the Vedic Mathematics plays an important role. Everywhere the Vedic Mathematics was received with much reverence and hopes. The Maths educators world over felt that the Vedic Maths is the wonderful system to develop the total brain of students and solve the basic problem of Maths anxiety being faced in Maths education the world over.

CONTINENTAL COURSES

International courses of two weeks each, on Vedic Maths were organised during June and July, 1988 at MERU Seelisberg, Switzerland, Mexico and M. I. U. IOWA, U. S A. All the lectures were delivered by Dr N. Puri, Reader in Civil Engineering, University of Roorkee. The MERU course was joined by participants from England. France, Spain, Holland, Germany and India. The U. S. A. and Mexico courses were joined by participants from Mexico, Panama, Canada and different states of America. 88 hrs of Video recording was done. These are Planned to be offered as future courses in different parts of the world. All the participants were highly impressed by the unique features of Vedic Maths. Several schools in USA etc. have started the teaching of Vedic Mathematics at different levels.

The initial feed back from different parts of the world is excellent. The participants of these courses have started delivering lectures and conducting courses in different parts of the world inclu-

ding China, Hongkong, Canada, USA, Japan, Sweden, France, Lebanon, England and Holland .

GLIMPSES :

1. NIKHILAM MULTIPLICATION

We have learnt the second Vedic sutra (Nikhilam Navatascaram Dastah) means all from nine last from Ten . Nikhilam method of multiplication can be used for multiplying numbers with big digits .

Example 1 : Multiply 9×6.

Step 1 : Select nearest base . Here base ia 10 . Base is the number with left digit as 1 and all other digits as 0, Therefore 10, 100, 1000, 10000 etc. are different bases .

$$\begin{array}{c} 9 - 1 \\ 6 - 4 \\ \hline 5 / 4 \\ \hline \end{array}$$

Step 2 : Compute the deviation of each number from the base , Write the deviation with sign in front of the number as shown. We use Nikhilam sutra . Deviation of 9 is 1 (last from ten 10-9). Since 9 is smaller than the base 10, we write this with the negative sign. Similarly, deviation of 6 is 4. (10-6=4) , which is the same as the purak of the last digit from 10 . Since 6 is smaller than the base, we write the deviation with a negative sign ,

Step 3 : Divide the result space by drawing a slash .

The right hand side of the result is simply the multiplication of the two deviations . Therefore $\bar{1} \times \bar{4} = 4$.

This is written on the right hand side side of the slash .

Step 4 : The left hand side of the result is simply the addition of any number with the deviation of the second number .

(i) Therefore $6 + \bar{1} = 5$ or

(ii) $9 + \bar{4} = 5$

$$\begin{array}{l} 9 - 1 \\ 6 - 4 \\ \hline 5 / 4 = 54 \\ \hline \end{array}$$

The resulting 5 is written on the left hand side ,

Hence 54 is the answer .

Example 2 : Multiply 8×9 .

Step 1 : Base is 10.

Step 2 : Compute the deviations .

$$\begin{array}{l} 8 - 2 \\ 9 - 1 \\ \hline 7 / 2 = 72 \\ \hline \end{array}$$

The puraks are $\bar{2}$ and $\bar{1}$.

Step 3 : Multlply the deviations $\bar{2}$ and $\bar{1}$ so $\bar{2} \times \bar{1} = 2$ (R. H. S.)

Step 4 : Cross and $8 + \bar{1} = 7$ or $9 + \bar{2} = 7$ (L. H. S.).

Hence 72 is the result .

Even subbases can be used (Refer AVM Pushp 2)

$$\begin{array}{l} 9\ 7 - 3 \\ 9\ 6 - 4 \\ \hline 9\ 3 / 12 \\ \hline \end{array} \qquad \begin{array}{l} 1\ 0\ 4 + 4 \\ 1\ 0\ 3 + 3 \\ \hline 1\ 0\ 7 / 12 \\ \hline \end{array}$$

2. MULTIPLICATION - THREE OR MORE NUMBERS

The superfast Vedic procedure of multiplication can be further extended to multiply three, four or even more numbers simultaneously. Of course, the procedure becomes just a bit longer for larger nnmbers. This is another very special and unique feature in the cap of the 'Nikhilam Navatascaramam Das atah' Vedic Sutra The following few examples shall serve the purpose of

clearly demonstrating the very simple, straight forward step by step procedure .

As we already know. 'Nikhilam sutra'means all from 9 and last from 10.

Type – I

Let us take a simple example of multiplying 98 × 97 × 96.

Step 1 : We write these numbers in the Vedic base system style. We get the deficiency of each number from the base by invoking the 'Nikhilam sutra' . Thereforc. we get

(The base is 100)

98 - 02
67 - 03
96 - 04
————
/ /

Step 2 : For three numbere the product shall have three parts, Therefore two inclined lines (//) are drawn to demarcate the three parts of the answer .

Step 3 : We vertically multiply the three deficit mumbers and the resulting number is the right hand portion of the answer In this case (–02) × (–03) × (–04) = – 24 = $\overline{24}$ is the required right hand portion. This is written down in the right hand space .

98 – 02
97 – 03
96 – 04
————
/ / $\overline{24}$
————

Step 4 : The middle portion of the answer is obtained as follows . We multiply two deficit numbers at a time by rotation. The summation

of these three small products gives the middle portion of our answer and is written within the two slashes ,

In the present example we get,

$(-02 \times -03) + (-03 \times -04) + (-04 \times -02) = 6+12+8=26$

$$\begin{array}{r} 98-02 \\ 97-03 \\ 96-04 \\ \hline /26/\bar{2}\bar{4} \end{array}$$

Step 5 : The left hand portion of the resulting product is obtained by the algebraic addition of the first number with the deficiency of the other two numbers .

In this example the L. H. S. is $98 + (-03) + (-04) = 91$,

The resulting number is written at the appropriate space .

$$\begin{array}{r} 98 - 02 \\ 97 - 03 \\ 96 - 04 \\ \hline 91/26/\bar{2}\bar{4} \\ \hline \end{array}$$

NOTE : The left hand portion of the product can also be obtained in several alternative ways. This, to mention again is the unique fecet of Vedic mathematics and each approach provides an easy check .

3. INTRODUCTION

The previously illustrated examples in chapter 6, were all solved by using the Nikhilam sutra ,

However, there are a few more corollaries arising out of the Nikhilam Sutra discussed in the previous chapter . Some of these have been presented in chapter 9 of Pushp 1 .

In mathematical astronomy we frequently require the multiplication of multipication of multiplier digits consisting entirely of nines

with other numbers (multiplicand). For multiplication of this type the subsutra Ekanyunena-Purvena is used,

Case I : Multiply 6×9.

According to the sutra the multiplicand has to be decreased by 1 (this gives the L. H. S.) and for the R. H. S. subtract the L. H. S. from the multiplier (get complements of nines)

Therefore $6 \times 9 = (6 - 1) / 9 - (6 - 1) = 5/4 = 54$

Illustrations

(i) $9 \times 9 = (9-1) / 9 (9-1) = 8/1 = 81$

(ii) $12 \times 9 = (12-1) / 9-11 = 11/\bar{2} = 10/10-2 = 10/8 = 108$

(iii) $13 \times 99 = (13-1) / (99-12) = 12/87 = 1287$

(iv) $19 \times 99 = 18 / (99-18) = 18/81 = 1881$

(v) $778 \times 999 = 777/222 = 777222$

(vi) $9853 \times 999 = 9862 / 0137 = 98620137$

COMMENT While obtaining the R. H. S of the answer by subtracting the L H.S. from the multiplier (9999) all the digits including zero must be written ($9999 - 9862$) = 0137 ,

(vii) $3728387 \times 9999999 = 3728386 / 6271613 = 37283866271613$

(viii) $12345678912 \times 999999999 = 12345678911 / 87654321038$ **OR**

$= 1234567891187654321088$

For details refer AVM Pushp 2.

4. DIVISION - NIKHILAM METHOD

INTRODUCTION

We have seen how NIKHILAM Method can be used for the multiplication of big numbers. Some other applications of the Nikhilam

Sutra in Vinculum numbers, Table etc. have also been discussed. It is interesting to learn that the Nikhilam sutra can also be used for the division. Like multiplication the Nikhilam method is most convenient when the divisor consists of the big digits. Bigger the digits easier is the solution by Nikhilam method,

Example : Divide 216 by 89.

89_/ 216

Step 1 : Using Nikhilam we modify the divisor.

Here MD = 1 1

Step 2 : We write as 89_/ 216
- 11

Step 3 : We draw a slash after 2 digits and number of the digits of the dividend from R. H. S. and the digits of divisor are numbered from L. H. S.

```
        3rd 2nd Ist
8 9_/ 2 / 1  6
1 1
Ist 2nd
```

Step 4 : Multiply the last digit of *D* (here 2) with the first and second digits of MD respectively and place the product below the previous digits (2nd and Ist column respectively) as follows

```
8 9_/ 2/1 6
1 1     2 2
     -------
      2/
```

Step 5 : On adding the digits of 2nd and Ist column we get the result.

```
8 9_/ 2/16
1 1     22
     -------
      2/3 8
```

So Quotient = 2

Remainder = 38

5. YAVADUNAM METHOD

There is yet another method of squaring. This upsutra reads as follows Yavadunam Tavadunikrtya Vargarca Yojayet which means whatever the extent of its deficiency lessen it to the same extent and set up the square of the deficiency (Refer Pushp 1 Chapter 9).

ILLUSTRATION 1. Calculate the square of the number 98.

Step 1 : First we select the base nearest to the number. Here 100 is the base.

Step 2 : We divide the result in two parts by drawing a slash

$$98^2 = L.H.S/R.H.S.$$

Step 3 : The deficiency of the number from the base is (100 - 98 = 02) = 02,

Step 4 : Now we subtract this deficiency from the number to get the *L. H.S.* as 98 - 02=96.

$$\text{So } 98^2 = 96 / \text{R. H. S},$$

Step 5 : For R. H. S. result we take the square of the deficiency. So the square of 02 is 04. Hence the result is $98^2 = 96/04 = 9604$. As there are 2 zeros in the base (100) we must have 2 digits on R. H. S.

ILLUSTRATION 2 : Calculate 996^2.

L. H. S. $996 - 004 = 992$

R. H. S. $(004)^2 = 016$

Hence $996^2 = 992/016 = 992016$

7. BEEJGANIT

In Vedic Mathematics Beejganit is the original name for algebra. Beej essentially means the seed and Ganit means Mathematics. Therefore, Beejganit should essentially be the seed of different facets of Mathematics, This is quite clear and obvious if one studies mathematics the Ancient Vedic way. Now we shall introduce ourselves to some interesting features.

Like decimal system (the name itself derived from 'Dasham' - meaning ten in Sanskrit) the algebra has also originated from India and in a recent research study swami S. A. Paramhansa has explained how even the name algebra was developed from original Beejganit (presented in the Jaipur workshop).

Let us consider a polynomial $2x^2 + 2x + 4$. If we substitute $x = 10$ (the deci) we get

$$200 + 30 + 4 = 234 .$$

Therefore, the coefficients 2, 3 and 4 are nothing but the digits with appropriate place value decided by the power of x. So far it is easy to see but Vedic Mathematics carries it ahead in a sweeping manner. Consider the polynomial $2x^3 - 3x^2 - 4x + 5$. Apparently, it is difficult to see the clear connection with arithmetic, but after learning the Vedic Vinculum system we can clearly see that this is just another way of writing number $2^-3^-4\ 5$ (putting $x = 10$). Further, it is not necessary that the coefficients of different terms of x must be all less than the base (here 10). For example

$12x^3 + 11x^2 - 5x + 4$ is equivalent to

$1x^4 + 3x^3 + x^2 - 5x + 4$ which in decimal system is

$12*1000 + 11*100 - 5*10 + 4 = 13154$ also

$1*10000 + 3*1000 + 1*100 - 5*10 + 4$
$= 131\bar{5}4 = 13054$

We can further see that x may have any value depending on the unit being used. For example

2 feet 5 inches can also be written as $2x + 5$ (here $x = 12$) ,
2 yard 1 feet can be written as $2x + 1$ (there $x = 3$) .

This opens up tremendous possibilities for furrher simplification and ease in different branches of Mathematics. Some of the interesting applications have already been expounded. (Ref. 1, 3, 4, 5, 10 etc,) Now we can elearly see, how the Vedic procedures of algebra are directly applicable in arithmetic and vice versa. Division by the Paravartaya sutra (No. 4) is one example. Some interesting applications are being presented at other places .

REFERENCES

1. "Vedic Mathematics", Swami Bharti Krishen Tirth Ji, Pub. M. L. B. D., Delhi .

2. "Ancient Vedic Maths Pushp-1" Narinder Puri, English and Hindi Pub. Spiritual Science Series (*SSS*), Roorkee .

3. "Ancient Vedic Maths Pushp-2",Dr. Narinder Puri, English and Hindi Pub. Spiritual Science Series, Roorkee .

4. Correspondence course on 'Vedic Maths'Part-1, Part-II and Part-III, Dr. N. Puri, Organised by Inst. of Vedic *Sc*. (IVS) of *SSG*, Roorkee.

5, "Triples", by K. Williams, Pub. by VM Res. Group. London.

6, Overview of Vedic Maths', Dr. N. Puri, Presented at a Series of International Workshops in 17 Countries, Pub. *SSS*, Roorkee,

7. Lecture notes of Int. Foundation Courses on Vedic Maths held at USA, Switzerland and Mexico; Dr. N. Puri. Pub. SSS, Roorkee ,

8. "Discover Vedic Maths", K. William, Pub, by VM Research Group, London,

9. "Vertically and Crosswise- Urdhva Sutra", A. P. Nicholas, J. R. Pickles and K. Williams, Pub. by VM Res. Group, London.

10. 'Introductory Lectures on Vedic Maths', A. P. Nicholas, J. R. Pickles and K. Williams, Pub. by VM Res. Group, London.

11. 'Notes of Workshop on Vedic Maths held, at Roorkee, Dr. N. Puri, pub. by SSG, Aug. 1987 .

12. Ancient Sidhant Shiromany-Pt - Leelavati; Bhaskaracharya pub. Chaukhamba Delhi (Hindi), English translation by Brooks .

13. 'The Ancient Sulbha Sutras' - Texts on the Vedic Geometry; Swami Satya Prakash Saraswati and Dr. V. J. Mati, Pub. R. K. S. S., Allahabad .

14. 'Ancient Vedic Mathematics - Pushp - 3' ; Dr. N. Puri. Pub. SSS, Roorkee .

15. Higher Applications of VM; Dr. N. Puri. Pub. SSS, Roorkee .

16. Unique Features of VM; Dr. N Puri, Pub. SSS, Roorkee .

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A NOVEL METHOD OF FACTORIZALION ENVISAGED BY JAGADGURU BHĀRATI KRISHNA TIRTHAJEE MAHARAJ

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According to Mrs Trivedi Jagadguru used to say that He has reconstructed the sixteen formulae from the Atharva Veda, after assiduous research and meditation for eight years. Obviously these formulae are not found in Atharva Vedā in their present form but they have been reconstructed on the basis of intuitive revelation from the materials scattered here and there in the Atharva Vedā. So Vedic Mathematics which was discovered and propounded by late Jagadguru was left in U. S A. in 1958 for publication. But due to ill health as it was not possible for him to go to U. S A. for correctionof proofs and personal supervision of printing the type script was finally brought back to India after this Mahāsamdāhi in 1960. Later on this was published by B. H. U Varānasi in 1965.

In the present article the application of Sutrās of Jagadguru to factorization will be discusesed. There are two sub-sutrās (i) आनुरूप्येण proportionately and ii) आद्यमाद्येनान्त्यमन्त्येन the first by the first and the last by the last.

In a simple case of quadratic expression of the type $ax^2 + bx + c$. split the middle coefficient into two such parts that the ratio of the first ceefficlent to that first part is the same as the ratio of that second part to the last coefficient. For example let us take the case $2x^2+5x+2$ in which the middle coefficient is 5 and the parts are (4 and 1), so

that (2 : 4 = 1 : 2). Now this ratio *i. e.* ($x+2$) is one of the factors and the second factor is obtained by dividiug the first coefficient by the first term of the first factor and the last by the last term. So the second factor is $2x^2/x + 2/2 = (2x+1)$. Again if the parts of the middle coefficients are (1 and 4) we have (2 : 1 = 4 : 2) and hence ($2x+1$) is one of the factors and the second is $2x^2/2x + 2/1 = (x+2)$, Again if we take $2x^2 + 5x-3$, the parts are (6 and 1) so that (2 : 6 = 1 : 3) hence the first factor is ($x+3$) and the second factor is $2x^2/x - 3/3 = (2x-1)$.

The 3rd *S*ubsutra is गुणित समुच्चय: समुच्चय गुणित: which means "The product of the sum of the coefficients in the factors is equal to the sum of the coefficients in the product", For example :

$$(x+7)(x+9) = x^2 + 16x + 63$$

$$(1+7)(1+9) = 8 \times 10 = 80 = 1 + 16 + 63 = 80$$

also $(x+1)(x+2)(x+3) = x^2 + 6x^2 + 11x + 6$

$$(1+1)(1+2)(1+3) = 2 \times 3 \times 4 = 24 = 1 + 6 + 11 + 6 = 24.$$

The 4th *S*ub-sutra is लोपन स्थापनाभ्यां which means "by (alternate) Elimination and Retention". Let us consider for example the homogeneous second degree expression .

(1) $E \equiv 2x^2 + 6y^2 + 3z^2 + 7xy + 11yz + 7zx$.

(i) First eliminate z and factorize the remainder by the आद्यम सूत्र

$$\text{So if } z=0,\ E \equiv 2x^2 + 7xy + 6y^2 = (x+2y)(2x+3y)$$

(ii) Similarly if $y=0$, $E \equiv 2x^2 + 7zx + 3z^2 = (x+3z)(2x+z)$

(iii) By the two sets of factors, fill in the gaps caused by making z and y equal to zero So $E \equiv (x + 2y + 3z)(2x + 3y + z)$

(2) $E \equiv 3x^2 + y^2 - 2z^2 - 4xy + yz - zx$

(i) when $z=0$, $E \equiv 3x^2 - 4xy + y^2 = (x - y)(3x - y)$

(ii) when $y=0$, $E \equiv 3x^2 - zx - 2z^2 = (x - z)(2x + 2z)$

(iii) By filling the gaps we have :

$$E \equiv (x-y-z)(3x-y+2z)$$

(3) $E \equiv 3x^2+xy-2y^3+19zx+28z^2+9xw-30w^2-yz+19wy+46zw$

(i) when $z=0$, $w=0$; $E \equiv 3x^2+xy-2y^2=(x+y)(3x-2y)$

(ii) when $w=0$, $y=0$; $E \equiv 3x^2+19zx+28z^2=(x+4z)(3x+7z)$

(iii) when $x=0$, $w=0$; $E \equiv 28z^2-yz=(7z-2y)(4z-y)$

(iv) when $z=0$, $y=0$; $E \equiv 3x^2+9xw-30w^2 = (x-2w)(3x+15w)$

Hence $E \equiv (x+y+4z-2w)(3x-2y+7z+15w)$

(4) $E \equiv 3x^2+7xy+2y^2+11zx+7yz+6z^2+14x+8y+14z+8$

(i) when $y=0=z$; $E \equiv 3x^2+14x+8=(x+4)(3x+2)$

(ii) when $z=0=x$; $E \equiv 2y^2+8y+8=(2y+4)(y+2)$

(iii) when $x=0=y$; $E \equiv 6z^2+14z+8=(2z+2)(3z+4)$

Hene $E \equiv (x+2y+3z+4)(3x+y+2z+2)$.

Now we shall discuss the factorization of cubics and Biquadratics by simple argumentation method. If by any method we are able to know Binomial factor of a cubic we are able to find out quadratic by आद्यमाद्येन... method . An easier derive will facilitated the operation, first and last term are written by आद्यमाद्येन method and the middle term is written by the गुणित समुच्चय ... method for example if we consider the cubic $x^3+6x^2+11x+6$ we see that S_o (sum of the coeffs. of the odd terms) $= S_e$ (sum of the coefficients of the even terms).

Hence $(x+1)$ is one of the factors. by आद्यमाद्यैन ... method x^3 and 6 are respectively first and last coefficients in the quotient. Also we have S_c (sum of all the coefficients) $= 24$ and as in $(x+1)$ the $S_c = 1 + 1 = 2$. we infer that S_c of quotiect is $= 24/2 = 12$. As the coefficient of the first and last terms are respectively 1 and 6 and their sum is $(1+6) = 7$. the coefficients of the middle term $= 12-7 = 5$. S_o the required quotient is x^2+5x+6.

Argumentation Method

We know that $(x+a)(x+b)(x+c) = x^2 + x^2\Sigma a + x\Sigma ab + abc$

Example (1) $x^2+6x^2+11x+6$.

Here $S_c = 24$ and t_1 (last term) $= 6$ whose factors are 1, 2, 3 or 1, 1, 6, But sum of the factors must be equal to 6 (the coefficient of x^2) the second group is rejected and the only alternative is to accept 1, 2, 3 group. By testing we see that $\Sigma a + 6$, $\Sigma ab = 11$, and $abc = 6$. Hence $E \equiv (x+1)(x+2)(x+3)$.

(2) $x^2-6x^2+11x-6$. Here $S_c = 0$ and the last term is -6 whose factors ate $-1, -2, -3$. So that $\Sigma a = -6$, $\Sigma ab = 11$, $abc = -6$.
Henee $E \equiv (x-1)(x-2)(x-3)$.

(3) $x^2+12x^2+44x+48$. Here $S_e = 105$ and $t_1 = 48$ whose factors are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

Since $S_o \neq S_e$ and $S_c \neq 0$ $(x+1)$ and $(x-1)$ are out of court and possible factors are $x+2$, $x+4$ and $x+6$ as $\Sigma a = 2 + 4 + 6 = 12$, $\Sigma ab = 8 + 24 + 12 + 44$,

Hence $E \equiv (x+2)\ (x+4)\ )\ (x+6)$.

(4) $x^3-2x^2-23x+60$, Here $S_e = 36 \neq 0$ and $t_1 = 60$ whose factors are 1, 2, 3, 4, 5, 6, 10.12, 15, 20, 30, and 60, But $60 = 3 \times 4 \times 5$

and when $x=3$, $E\equiv0$, and when $x=4$, $E\equiv0$ hence the factors are -3, -4 and 5 as $\Sigma a= -2$, $\Sigma ab = -23$. So

$$E=(x-3)\ (x-4)\ (x+5)$$

(5) $x^4+10x^3+35x^2+50x+24$, Here $S_o=S_e$ hence $x+1$ is a factor. Also $t_1=24$ whose factors are 1, 2, 3, 4, 6, 8, 12, 24 But we must have $\Sigma a=10=1+2+3+4$, also $\Sigma ab = 2+6+12+4+8+3=35$ and $\Sigma abc=6+24+12+8=50$

$$\text{Hence } E=(x+1)\ (x+2)\ (x+3)\ (x+4)$$

From above examples we see how powerfully these Vedic sutras given by late Jagadguru, work.

So far as what by I have thought over it, if these Sutrās be pra ctise the mathematicians of the modern world with confidense I can say that they will be quite uscful and helpful in solving many other difficult problems of higher mathematics.

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and, when $x=3$, $E=0$, and when $x=-4$, $E=0$ hence the fact is $x-3$, -4 and 5 as $\Sigma a=-2$, $\Sigma ab=-23$. So

$$E=(x-3)(x+4)(x+5)$$

(2) $x^4+10x^3+35x^2+50x+24$. Here $S_1=S_2$ hence $x+1$ is a factor. Also $x_1=24$ whose factors are 1, 2, 3, 4, 6, 8, 12, 24 but we must have $\Sigma a=10=1+2+3+4$ also $\Sigma ab=2+3+4+6+8+12=35$ and $\Sigma abc=6+24+12+8=50$

$$\text{Hence } E=(x+1)(x+2)(x+3)(x+4)$$

From above examples we see how powerfully these Vedic sutras given by Sri Jagadguru work.

So far as what I have thought over it, if these Sutras be programmed in the computers of the modern world with confidence I can say that they will be very useful and helpful in solving many other difficult problems of Computer mathematics.

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VEDIC MATHEMATICS FINDS NO LINK IN VEDAS

By

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No doubt each and every cultured person of the world knows the holy vedas . There are four vedas - Rig, Sama, Yajur and Atharva. The vedic tradition was originally oral. When the vedic period began is uncertain . But, it is expected that this period is 1600 or 1700 B. C. It is learnt that those days the memory of scholars/Rishies was aided by versification and by terse statement of important points they used to say them sutras .

Vedic Mathematics written by late *Ś*ankar*ā*ch*ā*rya *Ś*ri Bh*ā*rati *kr*ṣ*ṇa* Tirathaji Mah*ā*r*ā*ja of Govardhana Matha, Puri is a monumontal work in Mathematics . The sixteen sutras and subsutras propounded by swamiji of course, put forward divine device of calculating arithmetical and algebraical problems mentally . Their neat and ready application to practical problems covers various branches of Mathematics which includes arithmetic, algebra, plane and solid geometry, plane and spherical trigonometry, geometrical and analytical conics, Astronomy, differential and integral calculus etc. The short hand formula in the shape of sixteen sutras and a number of subsutras is a product of rigorus study and assiduous rescarch of swamiji for eight years in the forest surrounding Sringeri . Revcred swamiji used to claim that he had re-constructed the sixteen Mathematical formulae from the Atharva Veda and its Parisista , But, these formulae are not found in the present recension of Atharva veda .

Some people are of the opinion that these formulae of vedic Mathematics were actually reconstructed on the basis of intuitive revelation, from materidals scattered here and there in the Atharva Veda.

But, in the daily News Paper "Nav Bharat Times" from Patna on the 26th February 1988 there was an Article written by Sri Gunakar Mule on vedic Mathematics. In that Article he had tried his best to prove that with a view to make vedas popular the Sixteen Vedic Sutras had been written. We find that he is very much firm in his opinion that the sixteen sutras and subsutras of Vedic mathematics are, of course, mental creation of Swamiji and they have nothing to do with Vedas. He quotes Sri V. S. Agrawala's assertion in his General Editor's foreword of vedic Mathematics that the Sutras and Subsutras do not appear in the hither to known Parisista of Atharva Veda. Mr. Agrawala further admits that the style of language also points to their discovery by swamiji himself. It is obvlous that none of renowned Hindu Mathematicians Aryabhatha, Brahmgupta, Mahavirachary and Bhaskaracharya who were also known to be well-versed in vedas could not point out that vedas are rich in Mathematics. Mr. Mule feels that Swamiji has tried to establish the supremacy of Vedas in the field of Mathematics too by tagging his own creation in the name of Vedas.

I myself have gone through the four published Vedas-Rig, Same, Yajur and Atharva and their Parisistas. But, I have also failed to find any trace of the sixteen sutras and their corollaries of the Vedic Mathematics in any form and in any context in any of these vedas and their Parisistas. And, henee it is a duty to we people to trace the sixteen Vedic Sutras in the Vedas.

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COMPUTATIONAL ASPECTS OF NONLINEAR OPTIMIZATION PROBLEMS AND THEIR PRACTICAL APPLICATIONS

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ABSTRACT

This paper surveys in brief the available literature on optimization and tries to highlight the problems being currently faced in the numerical solution of nonlinear optimization problems. Practical use of nonlinear optimization techniques in solving real life problems arising in different fields is also briefly discussed.

1. INTRODUCTION

Mathematical techniques for finding the optimal value (the greatest possible value or the least possible value) of a function are called 'optimization technique's. The function to be optimized (maximized or minimized) may be a function of any number of independent variables. It may also be subject to certain constraints in the form of inequalities or equalities. Sometimes the number of functions to be optimized may be more than one.

Optimization problems arise in various disciplines such as engineering design, manufacturing systems, economics, business administration, physical sciences etc. In fact the newly developed optimization techniques are now being tried in every sphere of human activity where decisions have to be taken in some complex situations that can be represented by maethematical models.

Keeping in view practical applications, efforts have always been

there to develop simple, efficient, and reliable methods for solving different types of optimization problems. Methods of calculus for determining the maxima and minima of functions were developed to help solve real life practical optimization problems . However, it was soon realised that the utility of the methods of calculus in finding the maxima or minima of a funetion gets very much restricted when the function is a function of several unknown variables and is subject to a number of constraints in the form of inequalities, or when the function to be optimized is nondifferentiable . Since the mathematic al models of a wide variety of practical problems are of this nature, a necessity was felt for developing computational techniqes which are specifically suited for solving such types of nonlinear optimization problems. This lead to the systematic development of a subject which is now commonly known as 'Mathematical Programming' or 'Optimization Techniques'. The advent of high speed computing machines and the growing requirements of society for the optimal economic and efficient use of the available natural resources has given a further fillip to the development of this subject . Currently a vast amount of literature is available on the subject. However the available techniques are far from being adequate in helping efficiently solve a wide variety of practical real life problems which are being thrown up by the rapid technological, social and industrial developments which are taking place daily. Efforts are, therefore, being continuously made to develop more efficient, reliable and faster methods for solving such practical problems .

In this paper we have tried to briefly acquaint the reader with the current state of affairs and some of the possible emerging trends in this field .

2. SOME BASIC DEFINITIONS

Different types of practical problems lead to different types of methematical models for which optimal solutions have to be obtained. However the most general form of a single objective optimization problem is :

Maximize (or Minimize) $f(X)$, $X \equiv (x_1, x_2, \ldots, x_n)$ (2.1)

Subject to $g_j(X) \leqslant$ or $=$ or $\geqslant b_j$, $j = 1, 2, \ldots, m$ (2.2)

$x_1, x_2, \ldots, x_n$ are called 'unknown variables' or 'decision variables' and $X \equiv (x_1, x_2, \ldots, x_n)$ is called a 'decision vector'. The function (2.1) which is to be maximized or minimized is known as the 'objective function'. The set of inequalities (2.2) constitute the set of 'constraints' of the problem. The optimization problem (2.1 – 2.2) has n unknowns and m constraints, By the solution of the optimization problem we mean the set of values of the unknowns $x_1, x_2, \ldots, x_n$ which do not violate the constraints (2.2) and at the same time optimize the objective function (2, 1).

The branch of mathematics concerned with solving such type of optimization problems is known as 'mathematical programming'. If, in an optimization problem the objective function $f(X)$ as well as the constraint functions $g_j(X)$ all happen to be linear then it is called a 'linear programming problem (LPP)'. If the objective function and (or even one of the constraints is nonlinear, it is known as a 'nonlinear programming problem (NLPP)'. An optimization problem in which the objective function is not subject to any constraints is called an 'unconstrained optimization problem' otherwise it is called a 'constrained optimization problem'. An optimization problem is called an 'integer (all integer) programming problem' if all the

variables $x_1, x_2, \ldots, x_n$ are restricted to have integer values. It is called a 'mixed integer programming problem' when some of the variables are restricted to have integer values while integer restrictions are not imposed on other variables. An optimization problem in which the variables are allowed to have either zero or one value is called a 'zero-one programming problem'. Problems in which the number of objective functions (2.1) to be optimized is more than one are called 'multiobjective problems'.

The set S of points which satisfies all the constraints of an optimization problem is called a 'feasible set' or a 'feasible solution'. In a minimization problem if for $X^* \epsilon S$ there exists neighbourhood $N\epsilon(X^*)$ around X^* such that $f(X) \geqslant f(X^*)$, $X \epsilon S \cap N\epsilon(X^*)$, then X^* is called a point of local optimality (or 'local optimal solution'). If $X^* \epsilon S$ and $f(X) \geqslant f(X^*)$, $X \epsilon S$, then X^* is called a 'global optimal solution',

A nonlinear optimization problem may have one or more than one local optimal solutions. If the problem has a unique local optimal solution then it is also the global optimal solution. However, if the problem has more than one local optimal solutions then one or more of these may be global optimal solutions. In a linear programming problem a local optimal solution is also the global optimal solution. However, in a nonlinear optimization problem this is not always the case. A nonlinear optimization problem in which the local optimal solution is also the global optimal solution is called a 'convex programming problem'.

3. LINEAR PROGRAMMING PROBLEMS (LPP)

Linear programming is at present the most completely developed technique of nnmerical optimization. Linear programming techniques

are being widely used to solve practical problems in variety of fields such as economics, industry, transportation, social sciences, etc. A number of standard texts are available on this subject (see for instance Gass [47]). Simplex method developed by Dantzig [27] is still the most commonly used method to solve linear programming problems. Many practical problems formulated as linear programming problems have hundreds of constraints and huudreds of decision variables. These invariably have to be solved using digital computers. The first successful attempt on a first generation computer was made at National Bureau of Standards in early 1952. Further refinements in the simplex method were consequently carried out so that the implementation of the simplex method on a computer could be done more efficiently. Several versions of the simplex method are now available which can efficiently handle large size linear programming problems on computers. In 1984 Karmarkar [72] designed projective algorithm which can efficiently handle large size linear programming problems consisting of several thousand unknown variables and several thousand constraints.

Methods for solving special types of linear programming problems have also been developed. An important class of linear programming problems is called 'transportation or distribution problem'. The term 'integer programmiag' refers to the class of linear programming problems where some or all of the decision variables are restricted to be integers. Important developments in the earaly stages of the theory of LP are given in Dantzig [27]. Gale [46] and Hadley [57] give a mathematical treatment of simplex method from the view point of vector space theory. An excellent bibliography on the applications of LP in practice is available in Gass [47]. Gale [46] discusses many linear programming models of production, exchange and matrix games.

4. NONLINEAR PROGRAMMING PROBLEMS (NLP)

Whereas theoretically, every LPP can be solved by simplex method, there is on single technique which can claim to efficiiently solve each and every nonlinear optimization problem. In fact, a technique which is efficient for one nonlinear optimization problem may be highly inefficient for solving another nonlinear optimization problem. A variety of computational techniques have, therefore, appeared in literature for solving nonlinear optimization problems (see for instance Himmelblau [61], Bazaraa and Shetty [7], McCormick [84]) Most of these, however, determine at the most a local optimal solution. In the next few section we discuss in brief some of the impoitant methods for solving nonlinear optimization problems which are avaiable in literature.

5. METHODS FOR SOLVING UNCONSTRAINED NONLINEAR PROGRAMMING PROBLEMS

Unconstrained opiimization deals with the problem of minimizing (or maximizing) a nonlinear function of one or more more independent variables in the absence of any constraints. Several iterative procedures are available in literature for solving an unconstrained optimization problems. Most of these procedures involve a line search. In general, the efficiency of the line search method greatly affects the overall performance of the solution technique. For optimization of a nonlinear function of one variable some of the commonly used techniques are: the Fibonacci search procedure credited to Kiefer [71], the golden section method and bisection method etc. These procedures use only function evaluations at certain points in the region of search. Another class of procedures for the unconstrained optimizatio problems use curve fitting methods. If a function of one variable is to be minimized, the procedure involves finding an

approximating quadratic or cubic function and then minimizing it. Some limited computational studies on the efficiency of this approach are given in Himmelblau [61].

Among gradient free methods, the method of Rosenbrock [101] and the method Zangwill [123 (a)] are generally considered quite efficient for solving unconstrained problems of several variables. As originally proposed the Rosenbrock [101] method and the procedure of Hooke and Jeeves [64] do not use line search but employ instead discrete steps along the search directions. Incorporating a line search within the Rosenbrock's procedure was suggested by Davies, Swann, and Compey and is discussed by Swann [110].

There are yet other derivative free methods for unconstrained minimization of functions of several variables A procedure that is distinctly different is called the sequential simplex method. The method was proposed by Spendley Hext and Himsworth [108] and modified by Nelder and Mead [90]. The method essentially looks at the functional values at the extreme points of a simplex. The worst extreme point is rejected and replaced by a new point along the line joining this point and the centroid of the remaining points. The process is repeated until a suitable termination criterion is satisfied.

The method of steepest descent proposed by Cauchy in the middle of the nineteenth century continues to be the basis of several gradient based solution procedures. The method of steepest descent uses first order approximation of the function being minimized and usually performs poorly as the optimum is reched. On the other hand, Newton's method uses second order approximation and usually performs well at points close to the optimal. Several variations of Newton's method are also available. Recently Coleman and Conn

[25] have proposed some methods based on the second order derivative information .

Among the unconstrained optimization techniques, methods using conjugate directions are considered efficient . For a quadratic function these methods give the optimal solution in at the most n steps, n being the number of unknown independent variables in the problem. Some of the commonly used methods of this category are Powell [97*b*] and Zangwill [123*a*] . Some of the recent techniques which have appeared in this category are Dixon et al. [34], and Axeleson [6] .

Another class of methods, termed as variable metric or quasi-Newton methods, approximates the hessian matrix or its inverse using information from only first order derivatives . One of the early methods of minimizing a nonlinear function using this approach is that of Davidon [28], which was simplified and reformulated by Fletcher and, Powell [41] and is referred to as the variable metric method . A useful generalization of the Davidon-Fletcher-Powell method was proposed by Broyden [19]. Further details about variables metric methods are avilable in Spedicto [107] . For further details on methods for unconstrained nonlinear optimization one may refer to standard texts on the snbject such as Avriel [5], Bazaraa and Shetty [7] and McCormick [84] etc .

6. METHODS FOR CONSTRAINED NONLINEAR PROGRAMMING PROBLEMS

Several authors have tried to use unconstrained optimization methods for solving nonlinear problems with constraints . A successsful and frequently used approach is to define an auxilliary unconstrained problem such that the solution of the unconstrained

problem yields the solution of the constrained problem. Some authors have also modified the unconstrained optimization methods to handle constraints. Goldfarb [55] has extended the Davidon-Fletcher-Powell method to handle problems with linear constraints utilizing the concept of gradient projection. The method was generalized by Davies [29] to handle nonlinear constraints. Gill and Murray [50] have presented a survey on Newton type and Quasi Newton type methods for solving constrained problems.

Several gradient free search methods have also been extended to handle linear and nonlinear constrained problems Glass and Cooper [52] extended the method of Hooke and Jeeves [64] to deal with constraints. Box [15] developed a constrained version of the simplex method. Other alternative versions of the method were developed by Ghani [48], Umida and Ichikawa [115]. Another method that uses the simplex technique in constrained optimization was proposed by Dixon [33].

Another important class of methods used for solving constrained nonliear programming problems are known as penalty function methods. In penalty function methods the constrained problem is converted into an unconstrained problem or a sequence of unconsrt-ained problems in which there is a severe penalty for the violation of constraints. Use of penalty functions is generally attributed to Courant. However, significant progress in solving practical problems by the use of panalty methods followed the classic work of Fiacco and McCormick [38*a*].

Several extensions of the concept of penalty function have also been made. The work in this direction is still in progress (The

interested reader may refer to Fiacco and McCormick [38 *b*], Zangwill [123 *b*]).

Another class of methods for solving nonlinear optimizetion problems is the family of exact penalty function methods. The pioneering work in this field has been done by McCormick [84]. Work in this direction has also been done by Fletcher [40] and others. Recently Pillo and Grippo [96] have proposed a continuously differentiable penalty function for nonlinear programming problems.

Yet another class of methods used for solving constrained nonlinear optimization problems are known as methods of feasible directions. The method of feasible directions is a general concept that is exploited by a primal algorithm that proceeds from one feasible solution to another. One of the well known methods of this class is the feasible direction method of Zoutendijk [127]. Zangwill [123*b*] presented an improved version of a convergent algorithm based on the use of the concept of near-binding constraints. In 1960 Rosen developed the gradient projection method for linear constraints and later in 1961 generalized it for nonlinear constraints.

The method of reduced gradients was developed by Wolfe [119]. It was later generalized for nonlinear constraints by Abadie and Carpentier [1] who considered several approaches. Zangwill proposed the convex simplex method for solving a nonlinear programming problem with lenear constraints. The method could be viewed as a reduced gradient method. A comparison of convex simplex method with the reduced gradient method is given by Hans and Zangwill [59]. Recently Cheng [23] has given a dual gradient method for linearly constrained problems.

Keefer [70] proposed a method in which the basic search is the

Nelder and Mead simplex technique. The lower and upper bound constraints on the variables are dealt with explicity, while other constraints are handled by a penalty function scheme.

Paviani and Himmelblau [95] also use the simplex method in conjunction with a penalty function to handle constrained problems. The method is also known as flexible tolerance method. Flexible Tolerance method developed by Paviani and Himmelblau [95] uses only function evaluations to compute the optimal solutions of constrained as well as unconstrained optimization problrms. It combines the method of Nelder and Mead [90] with various additional rules to take care of equality and/or inequality constraints. The set of violated inequalities and equalities is lumped into one inequality constraint loosely satisfied during its initial stages. As the seareh proceeds the near feasibility limits are gradually made more and more restrictive until an acceptable feasible vector is obtained. In case of nonconvex problems the method eusures only a local optimal solution.

The sequential linear programming (SLP) approach for constrained oytimization problems finds the optimal solution by repeated linear approximation of a nonlinear problem and using linear programming techniques to solve it. These techniques are discussed in detail in McCormick 84] and Himmelblau [61]. Similarly, in sequential quadratic programming the basic concept is to create sequential quadratic approximations of the original nonlinear optimization problem. (For further details see for instance Biggs [11]. The latest discussions on this topic may de found in Conn [26]),

Sequential convex linearization is another approach [115] which has been tried. The basic concept of this approach is to first linearize the objective and the constraints as is done in SLP, However,

now instead of solving this linearized problem reciprocal variables are used to create a conservative convex approximation.

Attempts have also been made in literature to obtain the solutions of both constrained and unconstrained optimization problems by solving the Kuhn-Tucker conditions directly. Methods under this category are some times classified as multiplier methods (see for instance Arrow and Hurwitze [4]). Quasi-Newton type methods have been used in literature to obtain the solution to the systems of equations representing the Kuhn-Tucker conditions [13, *97a*]. Mohan and Tahir [87] also tried to obtain solutions to constrained and uncostrained nonlinear optimization problems by using a least square algorithm for solving a system of nonlinear equations representing the Kuhn-Tucker conditions of the problem.

Several studies for evaluating and testing nonlinear programming algorithms have been made. Computational results are reported in Himmelblau [61] using a range of nonlinear programming problems with varying degree of difficulty including highly nonlinear constraints and objective functions. For further details the interested reader can refer to Ragsdell 99] and Hung [65].

A class of nonlinear optimization problems which occurs frequently in problems of engineering design and which has been studied extensively in literature for quite sometime now is known as 'geometric programming problems' (see for instance Duffin, Peterson and Zener [36] and Beightler and Phillips [8]). The concept of geometric programming problems was later generalized to include the cases in which some of the cost coefficients may also be negative. Such problems are now known in literature as 'generalized geometric programming problems' (GGPP) or 'signomial geometric programming

problems, (SGPP). (c. f. Beightler and Phillips [8]). The geometric programming problems and signomial geometric programming problems are often solved using duality approach (see for instance Dembo [30], Beightler and Phillps [8]). Besides duality approach some other approaches have also been tried (for instance Mohan and Shashi Tyagi [88]).

In 1978 Lidor and Wilde [81] and Wilde [118] extended the concept of geometric programming problems to a more general class of nonlinear optimization problems which they called 'transcendental programming problems' (TPP) The concept of TPP was extended by Veena [116] to include the problems in which all the coefficients need not be positive and called these 'Signomial Transcendental Programming Problems' (STPP).

Several methods have also appeared in literature which are specifically suited for solving special types of nonlinear optimization problems. Mention in this regard may be made of the methods for solving Quadratic programming, Geometric programming, Separable programming. Dynamic programming, and Fractional programming problems. Discussion on these topics is available in standard texts such as Himmalblau [61], Beightler and Phillips [8]. Bazaraa and Shetty [7], McCormick [84] and Hartley [60] etc.

7. METHODS FOR DETERMINING THE GLOBAL OPTIMAL SOLUTION

In the absence of convexity, most of the methods discussed above at the most guarantee a local optimal solution. Keeping in view the practical necessity and the availability of fast computing machines, and computational techniques are now being reported in literature which try to search for the global optimal solution of nonlinear

optimization problems. Whereas Dixon and Szego [35*b*] give the status of the subject till seventies, Boggs [13] updates it to early eighties

The methods currently available in literature for solving global optimization problems may be broadly classified as deterministic methods and probabilistic methods. The deterministic methods try to guarantee that a neighbourhood of the global optima is attained. Such methods do not use any stochastic techniques, but rely on a thorough search of the feasible domain. They are applicable, however, to a restricted class of functions only such as, Lipschitz continuous functions, one dimensional functions, etc.

The trajectory method of Bramin [17] is one of the earliest deterministic methods. Treccani [114] showed that trajectory methods fail on some functions. Moreover these methods need analytic derivatives and of ten prove very expensive to use. Recently, Diener [32] proposed a new trajectory method but this too does not guarantee the global solution. Evtushenko [37] and Shubert [105] proposed grid methods for functions satisfying a Lipschitz condition. But these methods are not practical because it is not easy to find Lipschitz constant. Levy et al. [78*a*, 79] proposed a tunnelling algorithm for twice differentiable functions. Although the method generally succeeds in locating the global solution, it does not guarantee whether the global solution has been really found or not. In 1985 Levy and Gomez [78*b*] modified the Tunnelling algorithm and called it spherical tunnelling algorithm whidh they claim is better than the previous version.

In stochastic or probabilistic methods two phases are generally employed. In the first phase, or the global phase, the function is

evaluated at a number of randomly sampled points. In the second phase also called the local phase, the points are manipulated by local searches to yield a possible candidate for the global minima. Although probabilistic methods do not give absolute guarantee of success, these methods are sometimes preferred over the deterministic methods because they are applicable to a wider class of functions as they depend upon function evaluations alone nnd do not require the computation of derivatives etc.

Some of the earlier methods of this category are pure random search methods by Anderson [3]. Brooks [18] and Palosarri et al. [93]. Torn [113] proposed a clustering algorithm wbich was tested on a number of problems by Betro [9]. Boender and Rinnooy Kan [12] use Bayesian approach to solve global optimization problems. Dixon et al. [35*b*] snggested a method in which any local search method could be used to search the global solution by starting repeatedly ftom different initial points chosen stochastically.

Price [98] presented a controlled random search method in which the simplex approach is used on a random sample of points to yield a better point at each iteration and the method in the limit converges to the point of global minima. Recently, Mohan and Kusum Shanker [86] have updated the random search method of Price [98*b*] for solving the constrained nonlinear optimization problems. Shashi Tyagi [104] has developed a suitably modified version of random search method to solve integer and mixed integer programming problems.

8. MULTIOBJECTIVE AND GOAL PROGRAMMING PROBLEMS

Optimization problems in which it is desired to optimize more than one objective are called 'multiobjective' or 'vector optimization

problems' . Mathematical model of a multiobjective vector optimization problem is :

$$\text{Maximize or Minimize } [\, f_1 (X), f_2 (X) , \ldots , f_k (X) \,]^T \tag{8.1}$$

$$\text{Subject } g_j (X) \leqslant b_j , \; j=1, 2 , \ldots , m \tag{8.2}$$

Mathematical models of many real life optimization problems turn out to be multiobjective problems and these are now being extensively studied in literature (see for instance Zeleny [124], Changkong and Haimes [21], Steuer [109] and Zionts [126] etc.) . Multiobjective problems arise in different fields such as Game Theory, Networks, Markov Processes and engineering design (see for instance Golabi [54], Thomas [112 and Yu [120]) .

Many methods have been proposed in literature for solving multiobjective problems. Surveys and overview of multiobjective programming methods are given in Cohon et al . [24], Ignizio [66*b*] and Zionts [126] .

In a goal programming problem, it is desired to make the objective function achieve a specified value (called goal) as closely as possible rather than obtaining its optimal value . A goal programming problem may have a single objective or multiple objectives for whieh the goal/goals to be achieved are specified .

Goal programming was first used for linear problems by Charnes and Cooper [22*a* and extended by Iziri [68] and Lee [76]. In fact as they (Charnes and Cooper [22*b*]) point out later, they developed goal programming in the context of eonsttained regression when they attempted to find a formula for executive compensation in a division of the General Electric Company . There havs been numerous subsequent works on goal programming, particularly in the areas of

algorithmic development, applications and extensions (see for instance the survey by Karnbluth [73], texts on goal programming by Lee [76] and by Ignizio [66*a*] other references therein) .

Fuzzy set approach is now also being used in solving goal programming problems . In 1965 Zadeh [121] provided the first tool (fuzzy set) specially devised for dealing with different forms of imprecision, vagueness . By now over two thousand works dealing with this topic have been published. The theoey that Zadeh called fuzziness also stimulated a constructive debate on the several forms of uncertainty (Gaines [45]), and their mathematical rapresentation (Hohle and Klement [62] etc .).

Several authors kave used fuzzy set theory in solving goal programming problems . (see for instance Zimmermann [125], Hannan [58], Kacprzyk [69]) . They demonstrate how fuzzy or imprecise aspirations of the decision maker can quantified through the use of piecewise linear continuous functions. Shashi Tyagi [104] considered the effectiveness of solving multiobjective and goal programming problems using random search technique of global optimization as developed by Mohan and Kusum Shanker [86] .

9. SOLUTION OF PRACTICAL REAL LIFE PROBLEMS

Optimization teehniques, linear as well as nonlinear for single as well as multiobjective peoblems, find extensive practical use in various disciplines such as engineering design economics, business administration, physical sciences and manufacuring systems etc (see ior instance Bracken and McCormick [16], Beightler and Phillips [8], Himmelblau]61], Wilde [118], Osyczka [91] and Ozan [92] etc) . The work in this field is interdisciplinary in nature . In this section

we present a brief survey of available literature on topics in which we have been primarily working .

Gasoline blending problem, Circuit design problem, Foundry charging problem and Tolerancing mechanism components problems are solved using linear programming methods (c f. Oxan [92]). Mohan and Kusum Shanker [86] optimized the reliability of a complex system using random search techniqus , Paul and Tay [94] determined the optimal design of an Industrial refrigeration system . Sophos et al . [106] used goal programming in modeling petrochemical industry. Zanakis et ai . [122] solved chemical production planning problem via goal programming approach. Sakawa [102] solved multiobjective reliability design of a stand by system using sequential proxy optimization technique (SPOT) Grauer et al . [56] applied multiobjective criteria in chemical engineering . McCommon [83] apdlied goal programming approach in the design of Electro-acoustic transducers and transducer array . Nakayama et al . [89] used trade-off method in structural design .

Nonlinear optimization techniques have also been used for inversion of gravity data in geophysical prospecting (Al Chalabi [2], Fisher and Howard [39], Last and Kubik [75], Kusum Shanker, Khatri and Mohan [74*b*] . The problem of determining the location and the time of origin of an earthquake has also been solved as a nonlinear optimization problem (Gaiger [43], Bolt [14], Flinn [42], Gill and Murray [49], Dennis et al . [31], Gill et al. [51], Hong and Bond [63], and Kusum Shanker [74*a*]) .

Nonlinear optimization techniques also find extensive applications in structural engineering optimization. Venderplaats [117] considers the use of Numerical Optimization Techniques in Computer Aided

Optimal Design and Mechanical Systems. Minimum cost design of reinforced overhead water tanks has been considered by Rao and Raghavan [100], Chand et al. [20], Saxena et al. [103*a*, 103*b*], Templeman [111] optimized highway bridges using geometric programming. Saxena et al [103*c*] used nonlinear technigues of global optimization to obtain the minimum cost designs of the gas holders of biogas plants. Optimum designs of grid floor structures have been obtaincd by Goel [53 . Iyenger and Gupta [67] and Manu Saxena [82]. Manu Saxena [82] also used random search based technique of global optimization to obtain the minimum cost designs of reinforced concrete hanger type structures for storage godowns, industrial seeds and food grain storage silos. Some books on structural engineering have also appeared in literature. A comprehensive survey of structural engineering optimization techniques is available in Lev [77] . Levy and Lev [80] have reviewed recent developments in structural engineering optimization. Gaindhar, Mohan and Shashi Tyagi [44] have ussd nonlinear optimization methods to optimize riser design in metal casting. They have also considered the problem of determining the values of design parameters which can obtain desired values of characteristics in the mould in vacuum moulding process as a multiobjective noninear optimization problem [85] .

Bharati et al. [10] have used nonlinear optimization techniques to analyse the problem of town planning wiih particular reference to the prcblem of allocating the excess population of Delhi to various other towns of the National Capital Region .

10. EMERGING TRENDS

In view of the numerous practical applications of the optimization techniques and the fact that these can go a long way in proper

planning of production and efficient utilization of the available natural resources to meet the growing aspirations of the society, work is going on in this field at a rapid pace. Even though quite a large number of present day real life practical problems can be solved using the currently available techniques of optimization, there are still a large number of real life problems which cnnnot be satisfactorily solved (and some times even not solved) by the currently available techn-iques. Most of the currently available techniques of nonlinear optimi-zation can efficiently solve moderately sized nonlinear optimization problems on large mainframe computers. Eventhough linear program-miug problems having several thousand unknown variables and const-raints can be solved efficiently, there is on guarantee that a nonlinear optimization problem consisting of even a hundred or so unknown variables and constraints will be satisfactorily solved by available algorithms even on main frame computers. Efforts therefore being currently made to develop algorirhms which can efficiantly solve moderate sized nonlinear optimization problems on personal comput-ers (PC's) since practical users who are faced with moderate sized nonlinear optimization problems can at the moment ouly couveniently afford the facilities of PC's and not of large size compuing syftems Simultaneously efforts are also being made to develop algo-rithms which can solve large size nonlinear optimization problems on mainframe computers. Currently with the introduction of Super eomputers in which parallel processing is possible, cfforts are also being made to develop algorithms which make use of this facilitp. Efforts are also being made to develop algorithms which can solve nonlinear multiobjective goal programming problems in a more realis-tic and efficient manner as these problems more appropriately reflect the realities of true life situations.

11. CONCLUDING OBSERVATIONS

In this paper an effort has been made to briefly acquaint the reader with the current state of affairs in the field of nonlinear optimization techniques and their use in solving practical real life problems. However it is in no way claimed that the survey is exhaustive . There are several important methods and several important practical applications which have not been referred here . The interested reader can find reference to many such important works in the books and bibliographies of research papers given in references at the end of this paper .

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REFERENCES

[1] Abadie, J. and Carpentier, J., Generalization of the Wolf reduced gradient method to the case of nonlinear constraints, *Optimization*, R. Fletcher (ed.), 1969 .

[2] Al Chalabi, M., Interpretation of gravity anomalies by nonlinear optimization, *Geophysical Prospecting*, 20 (1972), 1-16.

[3] Anderson, R. S., 'Global optimization', in Optimization, R. S. Anderson, L. S. Jennings and D. M. Kyan (eds.), Univ. of Queenbland Press, 1972.

[4] Arrow, K. J. and Hurwitze, L., Reduction of constraint maxima to saddle point problems, *Proc. of Berkly Symp. on*

Mathematical Statistics and Probability, J. Neyman (ed.), Univ. of Calif. Press, 1065.

[5] Avriel, M., *Nonlinear Programming, Analysis and Methods*, Prentice Hall, Englewood Cliffs, N. J., 1976 .

[6] Axeleson, O,, A generalized conjugate gradient least square method, *Numerieche Mathematik*, **51**, No. 2 (1987), 209-228.

[7] Bazaraa, M. S. and Shetty, C. M., *Nonlinear Programming Theory and Algorithms*, John Wiley and Sons, 1979 .

[8] Beightler, C. S. and Phillips, D. T., *Applied Geometric Programming*. John Wiley and Sons, Inc., New York, 1976.

[9] Betro, B., Bayesian Testing of Non-paramertic Hypothesis and its application global optimization, *Jr. of Optimization Theory and Applications*, 42 (1984) .

[10] Bharati, Mohan, C., Garg, B. B. and Kusum Shanker., A Systems Analysis Approach to Town Planning. A Case Study of National Capital Region' paper presented in the *First Annual Conference of Vijnana Parishad of India*, Gurukul Kangri Univ. Hardwar. March 1989 .

[11] Biggs, M. C., Constrained minimization using recurrsive quadratic programming, some alternative subproblem formulations, *Towards Global Optimization*, L. C. W. Dixon and G. P. Szego (eds), North Holland Publishing Co. 1975 .

[12] Boender, C. G. E., Rinnoy Kan and Bayesian, A. H. C., Stopping rules for multistart globol optimization methods, *Mathematical Programming*, **37** (1987), 59-80 .

[13] Boggs, P. T., Byrd, R. H. nnd Schnabel, R. B. (eds.), Nume-

rical Optimization, *Proc. SIAM Conf. on Numerical Optimization*, Bouider, Colorado, 1985 .

[14] Bolt, B. A., The revision of earthquake epicentres, local depths and origin time using a high speed computer, *Geophysical J. of Royal Astronomical Society*, 3 (1960), 433-440 .

[15] Box, M. J., A new method of constrained optimization and a comparision with other methods, Computer Jr., 8 (1965), 42-52,

[16] Bracken, J. and McCormick, G. P., *Selected Applications of Nonlinear Programming*, John Wiley and Sons, Inc., New York, 1968 .

[17] Bramin, F. H., Widely convergent methods for finding multiple solutions of simultaneous nonlinear equations, *I. B. M. Jr. of Research and Development* (1972), 504-522 .

[18] Brooks, S. H., A discussion of random methods for seeking maxima, *Operation Research*, 6 (1958), 244-251 .

[19] Broyden, C. G., Quasi-Newton methods and their application to function minimization, *Mathematics of computation*, 21 (1967), 368-381.

[20] Chand, P., and Goel, B K., Computer Aided Designs of R. C. Beams by limit state method in accordance with IS : 456/1978, *SERC,, Ghaziabad, India*, 1987 .

[21] Changkong, V. and Haimes, Y Y., Multiobjective Decision Making : Theory and Methodology, *North Holland Series in Science and Engineering*, **11**, New York, Elsevier Science, 1983 .

[22 a] Charnes, A. and Cooper, W. W., Management Modeles and Industrial Applications of Linear Programming' *John Wiley and Sons, New York,* 1961.

[22 b] Charnes, A. and Cooper, W. W., Goal programming and constrained regression-a comment, *OMEGA* 3 (1975), 403-409.

[23] Cheng, Y. C., Dual gradient methods for linearly constrained strongly convex, separable, mathematical programming problems, *Jour. of Optimization Theory and Applications*, 53 (1987), 237-246.

[24] Cohon, J. L. and Marks, D. H., A review and evaluation of multiobjective programming techniques, *Water Resources Research,* **11** (2) (1973), 208-222.

[25] Coleman, T. F. and Cann, A. R., On the local convergence of a quasi Newton method for nonlinear programming problems, *SIAM Jour. of Numerical Analysis*, **21** (4) (1984), 755 .

[26] Conn, A. B., Nonlinear programmnig exact penalty functions and projection techniqes for non smooth functions, in Numerical Optimization, (eds.) P. T. Boggs, R. H. Byrd, and R.B, Schnabel, *Proceedings. SIAM Conference on Numerical Optimization , Bouider. Colorado,* 1985 .

[27] Dantzig, C. B., *Linear Programming and Extensions,* Princeton Univ. Press, Princeton New Jersey,, 1963 .

[28] Davidon, W. C., *Variable metric method for minimization,* AEC Research Development Report, 1959 .

[29] Davies, D., *Some Practical Methods of Optimization* in Integer and Nonlinear Programming, J. Abadie (ed.) 1970.

[30] Dembo, R, S. Current state of the art of algorithm and eomputer software for geometric programming, *Jr. of Optimization Theory and Applications*, 26 (1978), 149-183.

[31] Dennis, Jr. J. E. and Schnabl, R. B., *Numerical methods for unconstrained optimzation and nonlinear equations*, Princeton Hall, Englewood Cllife, N. J., 1983.

[32] Diener, I., Towards a trajectory method for globnl optimization, presented in the *Workshop on Global Optimization*, Sapron, Hungary, 1985.

[33] Dixon, L. C. W,, ACSIM An accelerated constrained Simplex technique. *Computer Aided Design*, 5 (1972), 23-32.

[34] Dixon, L. C. W., Ducksburg, P. G, and Singh, P., A new three term conjugate gradient method *Jr. of Optimization Theory and Applications*, 47 (1985), 285-300.

[35a] Dixon, L. C. W. and Szego, G. P. (eds), *Towards Global Optimization*, North Holland, Amsterdam, 1978.

[35b] Dixon. L. C. W. and Szego, G. P. (eds.), *The global optimization problem, an introduction in Towards Global Optimization* -2, North Holland, Amsterdam, 1978.

[36] Duffin, R. J., Peterson, E. L. and Zener, C., *Geometric Programming*, John Wiley and Sons, Inc., New York, 1967.

[37] Evtushenko, Y. P., Numerical methods for finding global extrema of a nonuniform mesh, *USSR Computing Machines and Mathematical Physics*, 11 (1971), 1390-1403,

[38a] Fiacco, A. V. and McCormick, G.P., Computatiou algorithm for the sequential unconstrained minimization techniques for nonlinear programming, *Management Science*, **10** (1964), 601-617 .

[38b] Fiacco, A. V, and McCormick, G. P., *Computational Algorithm for the Sequential unconstrained Minimization Technique,* (SUMIT), John Wiley and Sons, New York, 1968 .

[39] Fisher. M. J. and Howard, L. E., Gravity interpretation with the aid af quadratic programming, 45 (1980), 403-419 .

[40] Fletcher R., A l-penalty method for nonlinear constraints, in Numerical Optimization, P. T. Boggs, et al (eds.) *Proc. SIAM, Conf. on Numerical Optimization*, Colorado, 1984 .

[41] Fletcher, R. and Powell, M. J. D., A rapidly convergent descent method for minimization, *Computcr Journal*, 6 (1963), 163-168.

[42] Flinn, E. A., Local earthquake location with an electronic computer, *Bull. Seism., Soc. A*, 50 (1960), 467-470 .

[43] Gaiger, L., Probability method for the determination of earthquake epicenters from the arrival time only, *Bull, St. Louis Univ.*, 8 (1912), 60-71 .

[44] Gaindhar, J. L., Mohan, C. and Shashi Tyagi, Optimization of Riser Design in Metal Casting *Journal of Engg. Optimization, U. K*, 13 (1988), 1-26 .

[45] Gaines, B. R., Foundation of fuzzy reasoning, *Int. Jr. Man-Machine Stud.* 8 (1976), 623-668 .

[46] Gale, D., *The Theory of Linear Economic Models*, McGraw-Hill, New York, 1960.

[47] Gass, S. I., *Linear Programming Methods and Applications*, McGraw-Hill, New, York, 1985 .

[48] Ghani, S. N., An improved complex method of function minimization, *Computer Aided Design*, 4 (1972), 71-78 .

[49] Gill, P, E. and Murray, W., *Numerical Methods for Constrained Optimization*' Academic Press, London, 1979 .

[50] Gill, P. E and Murray, W , Algorithms for the solution of nonlinear least squares problem, *SIAM J. of Numerical Analysis*, 15 (1978), 977-992 .

[51] Gill, P. E , Murray, W. and Wright, M. H., *Practical Optimization.* Academic Press, New York, 1980 .

[52] Glass, H. and Cooper, L . Sequential search : a method for solving constrained optimization problems, *Jr. Association Computing Machinery* (1965), 71-82

[53] Goel, P. L. *Minimum Cost Design of Grid Floors*, M. E. Thesis, Department of Civil Engineering, Univ. of Roorkee, Roorkee, 1975.

[54] Golabi, K., *A Markov Decision Modeling Approach to a Multiobjective Maintenance* Problem. Lecture Notes in Economics and Mathematical System, No. 201, Berlin, Springer Verlag (1983), 115-125

[55] Goldfarb, D., Extension of Davidon's variable metric method to maximizing under linear inequality and equality constraints, *SIAM, Jr. Applied Mathematics*, 17 (1969), 739-764 .

[56] Grauer, M., Lewandowski, A. and Wierzbicki, A. P., Multiple objective decision analysis applied to Chemical Engineering, *Angewandte System Analyse*, 4 (1983), 32-40 .

[57] Hadley, G., *Linear Programming*. Addision Wesley. Reading, Massachusetts, 1962 .

[58] Hannan, E. L., Linear programming with multiple fuzzy goals, *Fuzzy Sets and Systems*, 6 (1981), 235-248 .

[59] Hans Tjian, T. Y. and Zangwill, W. T., Analysis and comparison of the reduced gradient and convex simplex method for convex programming, *ORSA, 41st Nat'l meeting, New Orleans*, 1972 .

[60] Hartley, R., *Linear Nonlinear Programming, Ellis Horwood Series in Mathematics and its Applications*, Wiley, New York, 1985 .

[61] Himmelblau, D. M., *Applied Nonlinear Programming*, McGraw-Hill, 1972 .

[62] Hohle, U. and Klement, E. P., Plausibility measures : a general frame work for possibility and fuzzy probability measures, In H. J. Sakala, S. Termini and E. Trilias (eds.) Aspects of Vagueness, D. Reidel, Dorderecht, 1984 .

[63] Hong, M. and Bond, L. S., Application of the finite difference method in Seismic Source and wave diffraction simulation, *Geophysical J. of Royal Astronomical Society*, 87 (1986), 731-753 .

[64] Hooke, R. and Jeeves, T. A., Direct search solution of numerical and statistical Problems, *Jr. Association Computer Machinery*, 8 (1961), 212-224 .

[65] Hung, M. S., Waren, A. D. and Lasdon, L. S., *Status of NLP software an update*, Joint National Meeting, Sheraton, Boston, 1985 .

[66a] Ignizio, J, P,, *Goal programming and Extensions*, Lixington Brooks, Massachuserts, 1976.

[66b] Ignizio, J. P., A review of goal programming : a tool for multiobjective analysis *Jr. of Operational Research Society*, 29 (11) (1978), 1109-1119.

[67] Iyengar, G. P. and Gupta, S. K., *Programming Methods in Structural Design*, East-West Press, 1980,

[68] Iziri, Y., *Management Goals and Accounting for Control*, Chicago, Rand McNally, 1965.

[69] Kacprzyk, J. and Oriovski, S. A., *Optimization Models using Fuzzy Sets and Possibility Theory*, D. Reidel Publishing Company, Dordrecht/Boston, 1987 .

[70] Keefer, D. L., SIMPAT : Self bounding direct search method for optimization, *Jr. of Industrial and Engineering Chemistry Products Research aad Development*, 12 (1973) .

[71] Kiefer, J.. Sequential minimax search for a maximum, *Proc. Amer. Math. Soc.* 4 (1953), 502-506 .

[72] Karmarkar, N., A New Polynnmial Time Algorithm for Linear Programming Problems, *Combinatorrica*, 4 (1984), 375-395 .

[73] Karnbluth, J. M. S., A survey of goal programming, *OMEGA* (1973), 193-205 .

[74a] Kusum Shanker. *Computational algorithms for Global Optimization problems and their applications with particular reference to Geophsics*, Ph. D. Thesis, Roorkee University, Roorkee, 1988 .

[74b] Kusum Shanker, Khatri, K. N. and Mohan, C., A random search technique of nonlinear optimization for inversion of gravity dsta, *VIIth Indian Geological Congress, Roorkee Univ., Roorkee*, 1987 .

[75] Last, B I. and Kubik, K., Compact gravity Inversion, *Geophysics*, **48** (1983), 713-721.

[76] Lee, S. M., *Goal Programming for Decision Analysis*, Auerbach, Philadelphia : Auerbach, 1972.

[77] Lev, O. E. (Ed.), *Structural Optimization Recent Developments and Applications*, Published by ASCE, New York, 1981 .

[78a] Levy. A. V. and Gomez, S, The turnelling method applied to global optimization, *Numerical Optimization*, P. T. Boggs, R. H. Burd and R. B. Schnabel (eds), SIAM, Philadelphia, 1985 ,

[78b] Levy, A. V. and Gomez, S., A spherical tunnelling algorithm for global optimization, presented in *Workshop on Global Optimization*, Sapron, Hungary, Dec 1985 .

[79] Levy, A. V. and Mantalova, A., The tunnelling algorithms for global optimization of functions, *SIAM*, *J*· of *Scientific and Statistical Computing*, **6** (19 5), 15-29 .

[80] Levy, R. and Lev, O. E., Recent Developments in Structural Optimizatton, *Jr. of Structural Engineering ASCF*, **113** 9 (1987) .

[81] Lidor, G. and Wilde, D. J., Ttanscendental geometric programs *Jr.of Optimization Theory and Applications*, 26 (1978) .

[82] Manu Saxena, *Optimum Designs of some Civil Engineering*

Structures using Nonlinear Optimization Techniques, Ph. D. Thesis, Roorkee Univ., Roorkee, 1988 .

[83] McCommon, D. F. and Thompson, W., The use of goal programming in the design of electro- acoustic transducers and transducer arrays, *Computer and Operations Research,* 10 (1983), 345-356 .

[84] McCormick, G. P., *Nonlinear Programming, Theory,* Algorithm and *Applications,* John Wiley and Sons, New York, 1983 .

[85] Mohan, C., Gaindhar, J. L. and Shashi Tyagi Optimization characteristics of shell lined diecasting (SLDC) process and castings made by SLDC process using interactive multiobjective goal programming and fuzzy set approaches. *Proceedings Nationai Seminar on Near NET-Shape Processes, Ranchi,* pp. 12. 01-12. 16, April 1989 .

[86a] Mohan, C. and Kusum Shanker, Reliability Optimization of Complex Systems using Random Search Technique, Microelectronics and Reliability, U. K . 38 (1988), 515-518 .

[86b] Mohan, C. and Kusum Shanker, A Numerical Study of Some Modified Versions of Controlled Random Search Method for Global Optimization Problems, *International Jr. of Computer Mathematics, U. K.,* 24 (1988) .

[87] Mohan, C. and Muzahim-Bani-Tahir, A least square algorithm for nonlinear optimization problems, *OPSEARCH,* 17 (1980), 81-92 .

[88] Mohan, C. and Shashi Tyagi, Use of global optimization techniques in solving signomial geometric programming problems.

Proceedings of the XIX Annual Convention of the Operational Research Society of India, New Delhi, 1986 .

[89] Nakayama, H. and Furrkawa, K., Satisfying trade-off method with an application to multiobjective structural design, *Large Scale System*, 8 (1985). 47-59 .

[90] Nelder, J. A. and Mead, R., A simplex method for function minimization, *Computer Jr.*, 7 (1964), 308-313 .

[91] Osyczka, A., *Multicriterion Optimization in Engineering with Fortran Programs*, Ellis Horwood Limited, John Wiley and Sons, New York, 1984 .

[92] Ozan, M. T., *Applied Mathematical Programming for Production and Engineering Management*, Prentice Hall. Englewood Cliffs, New Jersey, 1986 .

[93] Palosaari, S., Parviainen, S., Hiironen, J., Reunanen, J., Neittaanmaki, P., A random search algorithm for constrained global optimization, *Act a polytechnua, Scandinavica, Chemical Technology and Metallurgy Series No.* 172, Helsinki (1986), P. 45.

[94] Paul, H. and Tay, A. O., Optimal design of an industrial refrigeration system, *Proceedings of International Conference on Optimization Techniques and Applications, Singapore* (1987) 427-435 .

[95] Paviani, D. A. and Himmelblau, D. M., Constrained nonlinear optimization by heuristic programming, *Operations Research*, 17 (1969), 872-882 .

[96] Pillo, G. A. and Grippo, L., A continuously differentiable exact penalty function for nonlinear programming problems

with inequality constraints, *SIAM Jr. of Control and Optimization*, 23 (1985), 72 .

[97a] Powell, M. J. D., A fast algorithm for nonlinear constrained optimization calculations, *Proc. Dundee Conference on Numerical-Analysis*, 1977 .

[97b] Powell, M. J. D., An efficient method for finding the minimum of a function of several variables without calculating derivatives, *Computer Journal*, 7 (1964), 155-162 .

[98a] Price, W. L., A controlled random search procedure for global optimization, in *Towards Global Optimization*, Vol. 2; L C. W. Dixon and G. P. Szego, North Holland, Amsterdam, 1978 .

[98b] Price. W. L.. Global optimization by controlled random search, *Jr. of Optimization Theory and Appiications*, 40 (1983). 333-348 .

[99] Ragsdell, K. M., The utility of nonlinear programming methods for Engineering Design, *New Directions in Optimal Structural Design*, Atrek et al (eds.) 1984 .

[100] Rao, C. V. S. K. and Raghavan, T. M. S., Optimum Design of Intze Tanks on Shafts, *Inaian Concrete J* 56 (1982), 24-27 .

[101] Rosenbrock, H. H., An automatic mothod for finding the greatest or least value of a function, *Cpmputer Journal*, 3 (1960), 175-184 .

[102] Sakawa, M., Interactive multiobjective reliability design of standby system by seqential proxy optimization technique

(SPOT), *International Jr. of System Sciences.* 12 (1981), 687-701 .

[103a] Saxena, M. Sharma,, S. P. and Mohan, C., Use of nonlinear optimization techniques in determining the optimal design of Intze Tanks on Shafts, *Journal of Engg. Optimization, U. K.* 1985 .

[103b] Saxena, M.. Sharma, S. P. and Mohan, C., Cost optimization of Intze Tanks on Shafts using nonlinear programming, *Journal of Engg. Optimization,* 10 (1987), 279-288 .

[103c] Saxena, M., Sharma, S. P. and Mohan, C., Optimum of Gas holders of biogas plants, *National Workshop on Technology Delivery System, for Rural Development, New Delhi,* Oct., 1988 .

[104] Shashi Tyagi, *Optimization Techniques and Their Use in Solving a Class of Engineering Design Problems,* Ph. D. Thesis submitted to Roorkee Univ. Roorkee, 1988 .

[105] Shubert, B. O,, A sequential method seeking the global maxima of a function, *SIAM Jr. of Numerical Analysis,* 9 (1972), 379-388 .

[106] Sophos, A., Rotstein, E. and Stephanopoulous, G., Multiobjective analysis in modeling the pertochemical industry, *Chemical engineering Sciences,* 35 (1980), 2415-2426 .

[107] Spedicato, E.. Varinble metric methods for unconstrained optimization, *Nonlinear programming,* L. C. W., Dixon (ed,), 1980 .

[108] Spendly, W., Hext, G. R. and Himsworth, F. R , Sequential application of simplex designs of optimization and evolutionary operations, *Technometries,* 4 (1962), 441-461 .

[109] Steuer, R. E., *Multiple Criteria Optimization : Theory, Computation and Applications*, John Wiley and Sons, New York, 1986 .

[110] Swann, W. H., *Seport on the Development of New Direct Search Method of Opimization*, Chemical Industries Ltd. Central Inst. Res . Lab. Research Note 64/3, London, 1964 .

[111] Templeman, A. B., The application of geometric programming to the optimum design of bridge structures, *Proc. of PTRC Symp. London*, (1971), 125-130.

[112] Thomas, L. C., *Constrained Markov Decision Process as Multi-objeetive Problems*, In S Frenoh et al. (eds.), Multi-Objective Decision Making : Academic Press London, (1983), 77-94 .

[113] Torn, A. A. *A Search Clustering Approach to Global Optimiztion, in Towards Global Optimization*, 2, L. C. W. Dixon and G. P., Szego (eds.), North Holland. Amsterdam, (1978), 49-62 .

[114] Treccani, C., On the convergence of Bramin's method : A counter example, *Towards Global Optimization*, L, C. W. Dixon and G. P, Szego (eds.), North Holland, Amsterdam, 35 (1975) .

[115] Umida, T. and Ichikawa, A., A modified complex method for optimization, *Jr. of Industrial and Engineering Chemistry Product Research and Development*, 10 (1971), 236-243 .

[116] Veena Agarwal, *Geometric Programming and Transcendental Programming Problems in Nonlinear Optimization*, Ph. D. Thesis, Univ. of Roorkee, Roorkee, India, 1984 .

[117] Venderplaats, G. N., *Numerical Optimization Technique*, in computer aided optimal design and mechanical system, Carlos A. Mota Soares (eds) 1987 .

[118] Wilde, D. J., *Globally Optimal Design*, John Wiley and Sons. New York, 1978 .

[119] Wolfe, P., Method of nonlinear programming, in *Recent Advances in Mathematical Programming*, R. L. Graves and P. Wolf (eds.), 1963.

[120] Yu, P. L., Dissolution of fuzziness for better decision, perspective and techniques, *TIMS Studies in Management Sciences*, **20** (1984), 171-207 .

[121] Zadeh, L. A., Fuzzy Sets, *Inf and Control*, 8 (1965), 338-353 .

[122] Zanakis, S. H. and Smith, J. S., Chemical production planning via goal programming, *International Jr. of Production Research*, **18** (1980), 687-697 .

[123a] Zangwill, W. T., Minimizing a function without calculating derivatives, *Computer Journal*, **10** (1967), 293-296 .

[123b] Zangwill, W. T., *Nonlinear programming : A Unified Approach*, Prentice Hall, Englewood Cliffs, N. J., 1969 .

[124] Zeleny, M., *Linear Multiobjective Programming*, New York, Springer-Verlag, 1974 .

[25a] Zimmermann, H. J., Description and optimization of fuzzy systems, *Int Jr.* of *Gen. Syst.* **2** (1976), 209-215 .

[125b] Zimmermann, H. J., Fuzzy programming and linear programming with several objective funotions, *Fuzzy Sets Systems*, **1** (1978), 45-55 .

[125c] Zimmermann, H. J., Using fuzzy sets in operational research, European J. of Operatioual Research, **13** (1983), 201-216 .

[126] Zionts, S., Multiple criteria mathematical programming : An update overview and several approaches, *Vienna Proceeding Paper*, Vol. 821558001, Springer, V. 1987 .

[127] Zoutendijk, G. *Methods of Feasible Directions*, Elsevier, Amsterdam, 1960 .

Proc. VPI, Vol. 1, 1989

MATHEMATICAL AND COMPUTER BASED NUMERICAL APPROACH TO PHYSIOLOGICAL HEAT TRANSFER PROBLEMS

By

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1 INTRODUCTION

All living animals producc heat, which is either migrated to the surroundings or stored in the body itself. All mammals and most birds use this heat to maintain the internal parts of body at constant temperature. Heat is continuously being generated within the body as a result of chemical processes associated with breathing, digestion and physical efforts. The most important factor responsible for heat generation in a human body is metaboliam. The metabolic reactions take place in body cells. The rate of metabolic heat generation usually depends on environmental conditions.

Heat removal from an animal body is also a continuous process like heat generation. But in heat generation almost all regions of the body are involved while the migration of heat to the surroundings takes place only through the skin and respiration, However, the heat lost through respiration is not much and the quantitiy lost is easily made up by the dense system of blood capillaries present in the pulmonary region. The exchange of heat through peripheral regions is controlled by various body organs and physical conditions. The complexity of region makes this process very interesting and most of the physiological heat transfer problems are concerning with heat transfer through skin and subcutaneous tissues (SST). Any unified study of such problems has to take into account various factors

like metabolic heat generation, micro-circulation, perspiration, structure of the region with its complex physical parameters and environmental conditions.

Till recently, there has been hardly any analytical study of physiological heat transfer problems with an idea of providing general useful relationships. Some initial and specialized studies in allied aspects include the works of W. perl (J. Theo. Biol..2 (1962), 201), G. J. Trezek and T. E. Cooper (Therm. Prob. Bio. Tech. ASME, 1968, 1), V. P. Saxena (Proc. VI Heat Tr. Conf. 3 (1978). 65) and J. Stekete and M. J. Van Der Hoek (Phys. Med. Biol., 24 (1979), 31). Some papers on heat transfer problems in SST region of human body for some simple and moderate conditions have been brought out by V. P Saxena (Proc. XI Nat. Conf. Fl. Mech. Fl. Power, 2 (1979), 156), V. P. Saxena and D. Arya (Num. Methods in Thermal problems, Editor : Morgan and Lewis, Pineridge Press U. K.), V, P. Saxena and D. Arya (J. Theo. Biol., 81 (1981), 423), V. P. Saxena and J. S. Bindra (Proc . Symp. Math. Mod. M. R. I. Alla. (1982)), D. Arya and V.P. Sexena (Proc. Symp. Math. Mod. M. R. I. Alla. (1982).

Some recent work by Saxena (J. Theo. Biol. 102 (1983), 277) and Saxena and Bindra (Ind. J. Pure and Applied Math. 15 (1984), 31, 18 (1987), 846) presents total study of normal heat flow in dermal layers by exact as well as numerical approach. Heat regulation in body limbs is already under study at present, by Saxena and Bindra (Int. J. Math. and Math. Sci (1989)).

2. Mathematical Formulation.

The following equation governing heat transfer in skin and subcutaneous tissues has been obtained by making use of Fourier's laws and Fick's well known principle.

$$div\ [\ K(\bar{x}, u)\ \text{grad}\ u(\bar{x}, t)\ ] + m(\bar{x}, t)^{c_b}\ \{u_b(\bar{x}, t) - u(\bar{x}, t)\}$$

$$+ S(\bar{x}, u) = P(x)\ C(\bar{x})\ \frac{\partial u}{\partial t}(x, t) \qquad ...(1)$$

and $$K\frac{\partial u}{\partial n} = h(u - u_a) + LE \qquad ..(2)$$

at the skin surface

where

$K=$ thermal conductivity of the tissue

$m=$ mnss blood flow rate

$c_b=$ specific heat of blood

u_b = blood temoerature

$S=$ rate of metabolic heat generation

$p=$ tissue density,

$c=$ specific heat of the tissue

$u(\bar{x}, t)$ = temperature of the tissue at any position $\bar{x}$ and time t,

$\bar{x}=$ space variable,

$t=$ time variable,

$h=$ heat transfer coefficient

$u_a=$ atmospheric temperature

$L=$ latent heat of evaporation

$E=$ rate of evaporation

$\frac{\partial u}{\partial n}$ = partial derivative of u along normal to the surface .

The terms on the left of (1) are Fickian diffusion, perfusion and metabolic heat generation respectively. Various authors have used special cases of the above equations. Due to composite behaviour of the medium and irregular geometry of the subregions we have considred following variational form of the governing equations for heat flow in dermal and other peripheral parts of a human body :

$$I=\frac{1}{2}\int_{P} [k_x (u, x)^2 + k_y (u, y)^2 + k_z (u, z)^2$$

$$+ M c_b (u_b - u)^2 - 2Su] \, dP$$

$$+ \frac{1}{2} \int_{Q} [h (u-u_a) + 2 LEu] \, dQ$$

where Q is the boundary of the region P under consideration. This region can be divided into natural and convenient subregions depending on the variation of vorious parameters like tissue structure, tissue thermal properties, blood mass-flow rate and rate of metabolic heat generation. The variational finite element method provides flexibility of local considerations. In most our problems we have exploited this advantage of FEM and obtained solutions under various practical situations.

The same model in polar cylindrical co-ordinates has been employed to human limbs. However, to avoid cumber some computation and to save computer time we have modified FRM approach and deviced, what we call as Psuedo Analytic Finite Partition Method for circular cross sections. Assuming annular layered structure of the limbs and unsymmetical situation of blood vessels, the FRM technique has been used only in radial direction. The resulting system of differential

equations have been solved by using analytical methods such as Fourier series and Laplace transforms .

Recently we have taken up work related to abnormal physiological situations . At present our main attension is on the growth of malignant tumours in dermal regions and its effect on surface and internal tempernturеs. We have incorporated the effect of ever increasing uncontrolled metabolic activity of the tumour and changing profile of blood perfusion. This includes the nacrosis taking place at the core (Saxena and Pardasani; Int.Conf. Biomath. Xian. China 1988). (Pardasani and Saxena : Biomechanics, ed by Sahay and Saxena, published by Wiley Eastern, New-Delhi. 1988),for two dimensional traigular shape functions approximating main and tumour parts. A computer software has been developed for the same and ICIM-6000 machine has been used to obtain numerical results.

equations have been solved by using analytical techniques as well as [illegible] and [illegible] techniques.

Recently we have taken up work related to [illegible] biological situations. At present we are attempting [illegible] the growth of malignant tumours in different regions and its effect on surface and internal temperatures. We have incorporated the effect of [illegible] using [illegible] variable [illegible] of the tissues and [illegible] profile of blood perfusion. This [illegible] taking place at the [illegible] (Saxena and Pardasani, Int. Conf. [illegible] 1990). (Pardasani and Saxena [illegible] edited by Sahay and Saxena, published by Wiley Eastern, New Delhi 1989), for one-dimensional [illegible] shape functions [illegible] main and tumour parts. A computer software has been developed for the standard ICIM-6040 machine [illegible] has been used to [illegible] numerical results.

Proc. VPI, Vol. 1, 1989

AREA OF POLYGON

By

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1. Area in Terms of Sides of Polygon:

$$\text{Area} = \frac{na^2}{4} \cot \frac{\pi}{n}$$

where a is the side of polygon of n sides.

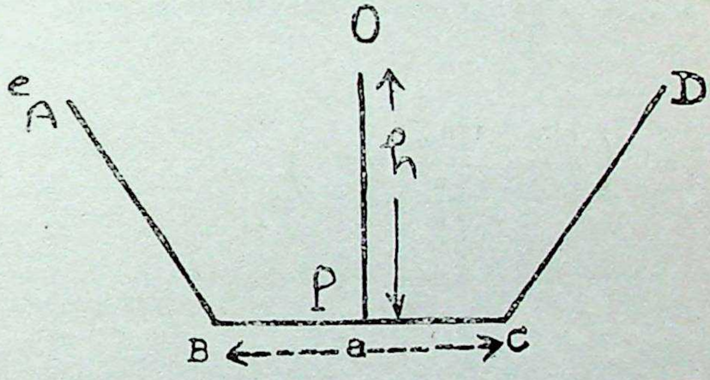
Derivation: Let ABCD......be a polygon of n sides with side length a. Let O be the centre of the polygon. Let h be the length of the perpendicular from O on one side (Fig 1). Now

$$\angle \text{BOP} = \frac{\pi}{n}.$$

From $\triangle$ BOP we have

$$h = \text{BP Cot BOP}$$

$$= \frac{a}{2} \text{Cot } \frac{\pi}{n}.$$



Therefore area of $\triangle$ AOC

$$= \frac{1}{2} a \times \frac{a}{2} \text{ Cot } \frac{\pi}{n}$$

$$= \frac{a^2}{4} \text{ Cot } \frac{\pi}{n}.$$

Hence Area of Polygon

$$= \frac{na^2}{4} \text{ Cot } \frac{\pi}{n} \qquad \text{......} (1)$$

2. Area in Terms of h.

From $\triangle$ BOP

$$\frac{a}{2} = h \tan \frac{\pi}{n}$$

or $$a = 2h \tan \frac{\pi}{n}.$$

Putting this value of a in (1) we have

$$\text{Area of Polygon} = nh^2 \tan \frac{\pi}{n}. \qquad \cdots\cdots (2)$$

3. Deduction of Formula for Area of Circle:

If we take $n \to \infty$ then polygon will become circle. In this case $\lim\limits_{n\to\infty} h = r$ (the radius of circle).

Thus area of circle

$$= \lim_{n \to \infty} nh^2 \tan \frac{\pi}{n}$$

$$= \lim_{n\to\infty} \frac{\pi h^2 \tan \frac{\pi}{n}}{\frac{\pi}{n}}$$

$$= \pi r^2 \left(\lim_{n\to\infty} \frac{\tan \frac{\pi}{n}}{\frac{\pi}{n}} = 1 \right).$$

Proc. VPI, Vol. 1, 1989

MATHEMATICS AND ITS APPROACH TO TECHNOLOGY

By

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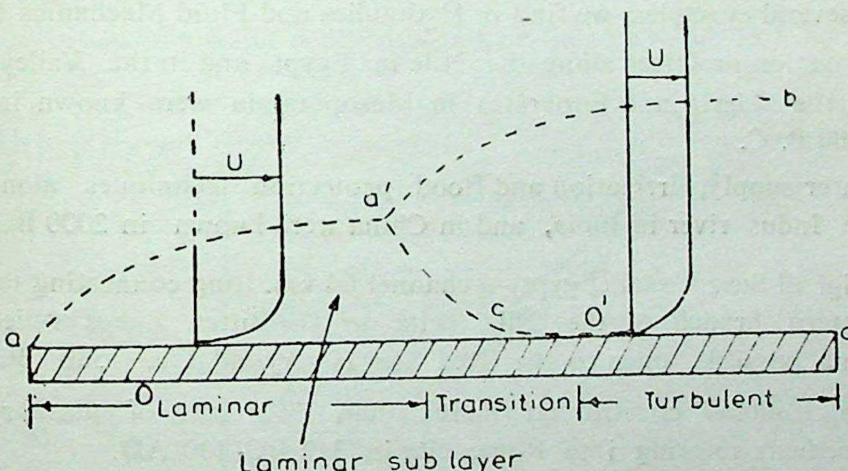
Mathematics as we study from the lower level to the higher level, changes its nature of study, apalications etc. For example in elementary Algebra, we solve some equations say

$$3x+y=2$$
$$x+4y=8$$

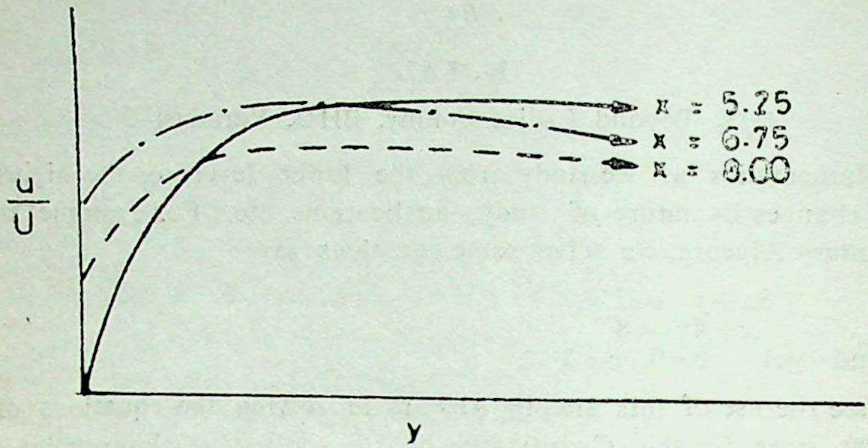
and get $x=0,\ y=2$

Now see the use of this simple Algebra in solving the equations of straight lines for the Coordinates of the points of intersections. This we study in early stage and it is said to part of Coordinate geometry. In higher study we draw graph say for the variations of some parameters, simplest the flow between parallel plates, say velocity and y distance. These are situation, where such curves intersect and at the point of intersecticn, we have equal values of velocity and y distance. The flow we know is divided into three parts i.e. laminar, transition, turbulent (1)



LAMINAR FLOW IS BETWEEN a a' c d AND THE SURFACE OF THE PLATE

Diagram for boundary layers on a flat plate.



Mean velocity profiles in the transition region on a flat plate

Above graphs are given to show one example where we solve the problem by considering the point of intersection and important conclusions are drawn.

Another example, we may take of flow through the open channel which we come across every day. This area is of practical applications, several evamples, we find in Hydraulics and Fluid Mechanics (2)

(1) Irrigation practices along the Nile in Egypt and in the Valleys of the Tigris and Euphrates in Mesopotamia were known in 3000 B. C.

(2) Water supply, irrigation and flood protection techniques along the Indus river in India, and in China were known in 2000 BC.

(3) Original Suez Canal (Egypt)–a channel 64 km. long connecting the eastern branch of the Nile delta to the Bitter Lakes which were possibly open to the Red Sea is very old i. e. 1500 BC.

(4) The Romans constructed more than 560 km. of masonry apueducts to bring into Rome city in 312 BC-130 AD.

This shows that mathematics and its applications are quite old. We know the use of simple mathematics in such problems. We use certain formulae in such cases as such:

(i) Chezy's equation (French Engineer 1775), given by

$$V = C. (RS)^{1/2}$$

where V = Velocity

C = Chezy's coefficient

R = hydroulic mean radius = A/P

A = area of cross-section of flow

P = wetted perimeter (wetted length of the chennel boundary)

S = Bed slope of the channel.

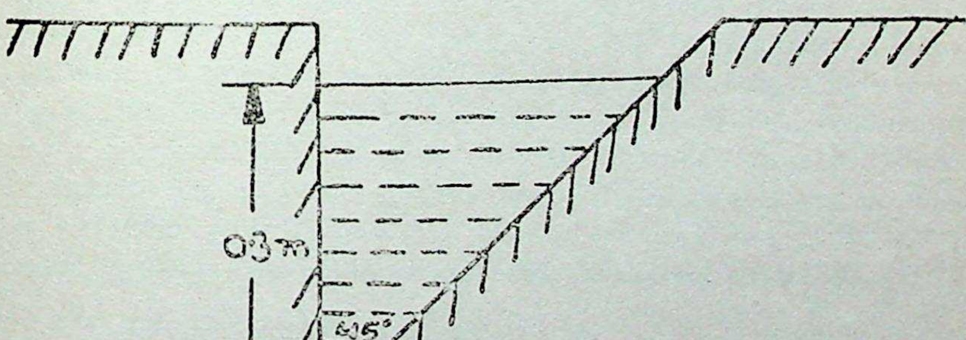
(ii) Manning formula (after Irish Engineers, 1891)

$$V = 1/n \; R^{2/3} \; S^{1/2}$$

where n = Manning's roughaess coeffecient.

Now we give one example of practical application which we see even in Haridwar the road side. (Ref2, P. 467).

A triangular road side gutter has one side vertical and the other side is at 45^0. It is laid at a slope of $\frac{1}{2}$ percent. Determine the storm flow when the water is 30 cm. deep. Assume $n = 0.028$



$$Q = AV$$
$$= \left(\tfrac{1}{2}\times 0.3\times 0.3\right)\ \frac{1}{0.028}\left[\frac{\tfrac{1}{2}\times 0.3\times 0.3}{0.3+((0.3^2(0.3)^2)^{1/2}}\right]^{2/3}.$$
$$.(0.005)^{1/2}$$
$$= 0.018 \quad \text{cumec} = 18 \text{ lit per second (lpc)}$$

These are several examples like above one where we use simple geometry of high shool to calculate as in above figure, area of the cross-section, length of the side of a triangle etc. The august gathering here is actively engages in higher research in various branches of Mathematics and they all know the use of Mathematics in Technology, Medical Sciences, Social Sciences, Political Science, Jyotish Sciences etc.

REFERENCES

1. J. G. Knunsen and D. L. Matz, Fluid Dynamics and Heat Transfer (Chapter 10).
2. H. M. Raghunath, Fluid Machanics and Machinary (1987).

Proc. VPI, Vol. I, 1989

S. RAMANUJAN- A Historical Perspective

By

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Mr. Chairman and Honourable Delegates:

It is a pleasure to be here on the occassion of first Annual Conference of Vijnana Parishad of India and Symposium on Vedic Mathematics-Traditions and Applications.

In 1987 we celebrated the centenary celebrations of great son of India S. Ramanujan and I think it would be in best Vedic traditions (though not in the from of Shlokas and Mantras) to pay homage to him who in a way incarnated a few qualities of Vedic Rishis.

SRINIWAS AIYANGAR RAMANUJAN AIYANGAR was born on December 22, 1987 at Erode a suburb on the banks of river Kaveri. His father was a Munshi at a Cloth merchant shop.

His mother was a pious lady and a great devotee of Goddess Namgiri. He regarded his father as Shiromani (best) of all males and his mother as incarnate of mother Parwati. Super criative activity in him was both intelect and intuition combined which he took to be a gift of Goddess. He passed his School leaving certificate examination in 1903 in first division. In 1909 he was married to one Janaki Devi. Now the worry of bread and butter made him to search for a suitable job as he could not fare well at F. A. examination. With the efforts of V. Ramaswami Aiyer (founder of Indian Mathematical Society in 1907), P. V. Sheau Aiyar and S. Narayan Aiyar he was able to get the post of a clerk in Madras Port Trust but to supplement his earnings he had to take to private tutions also.

Although, as mentioned above, he failed in F. A. examination he had acquired proficiency in Indeterminate forms in third standard, Loney's trigonometry in fourth standard but he was initiated into higher mathematics from a simple book "A Synopsis of Elementary Results in Pure and Applied Mathematics" by George Carr.

January 26, 1913 was Makar Sankrati an auspicious day for Hindus. This day proved to be auspicious too for modern Indian Mathematics. On this very day on persuasion and inspirtaion from P. V. Sheshu Aiyer S. Ramanujan wrote a long letter to Prof. Hardy of Trinity College England describing among other things his work on divergent series and integrals. Before coming into contact with Prof. Hardy Ramanujan had not seen any standard book on Elliptic functions. This is evident from the fact he did not recognise many characteristic properties of elliptic function like addition theorem etc. Also he used his own notations.

As it would have it Indian Mathematical Society was founded in 1907 by V. Rama Swami Aiyar and his group consiting of R. P. Prajpe, M. T. Narayan Aiyangar among others. Again in 1909 Ashutosh Makerji was instrumental in shaping Calcutta Mathematical Society and in 1919 Ganesh Prasad founded Banaras Mathematical Society (Presently Bharata Ganita Parishad Located at Lucknow University at present). These societies started three journals the Journal of Indian Mathematical Society, Bulletin of Calcutta Mathematical Society, and Proceedings of Banaras Mathematical Society (at present "GANITA"). Thus the history of modern Indian Mathematics begins with last half of first decade of this century. It was within this period that Ramanujan's genious stated showing off. His first paper enititled "Some properties of Bernoulli's Numbers" was published in the Journal of Indian Mathematical Society in 1911. (Vol. III December No .), though one of his earlier papers (in the form of questions) had appeared in Vol. III February number. Two more appeared in 1912.

Coming back to biographical sketch, it is aignificant to note that on receipt of previously mentioned letter from Ramanujan Prof. Hardy tried hard to persuade him to come to Cambridge. He deputed Prof. E. H. Neville for the purpose. And on March 17, 1914 they proeeded to Cambridge. However prior arrangements had to be made for supplementing, as desired by Ramanujan, the poor pecuniary condition of his parents (in the form of Rs. 60/- p. m. out of his scholarship to be paid to his parents) and by instructing streamer agents of S. S. Nevasa for providing strictly vegetarian food through

out the voyage.

During the five odd years of stay at Cambridge Ramanujan published 21 papers in different journals of Europe of which five were in Collaboration with Hardy.

As mentioned earlier before going to England he did not use the familiar Legendre k, k' K, K', E E', etc. neither he used Jacobian constant q or the Weierstrassian g_2, g_3, e_1, e_2, e_3, etc. It was only in his papers "Modular equations and approximations" and "On certain arithmetical functions" he uses the modern notations. He was so modest in nature that while proving his basic identity he writes "The elementary proof of these formulae given in the preceeding sections seem to be of some interest in itself". He proceeds to give a very simple proof of the Discriminant identity of Jacobi. It may be noted that the proof of Ramanujan's differential equation is entirely algebraic in character as also his other developments. The most intricate part of Ramanujan's work is the modular equations given in various forms and the evaluation of the corresponding singular modulii.

On February 28, 1918 (again on an auspicious day–MAHASHIVA-RATRI to be particular) Ramanujan was elected a fellow of Royal Society – the highest honour then in scientific world. He was (contrary to popular belief) second Indian *F. R. S.*, the first being Ardeseer Cursetjee) who was elected in 1847. On October 13, 1918 (during Dussehara) Ramnujan was elected a fellow of Trinity College, the first Indian to receive this honour. During this period Madras University (from 1st April 1919 to be exact) granted an yearly allowance of Pounds250 for him which he gratefsully acknowledged and wished that out of the amount "Pounds60 an year (be paid to my parents and that the surplus be used for poor boys and orphans". This shows his parents and havenots.

He had a great affection for his wife too. According to his personal friend Prof. G. C. Chatterjee he felt morose and sad if he did not receive regular letters from his wife. Prof Neville recalls " The wonderful mathematician was a lovable man. " He was a good conversationalist and a better companion. He was quite simple and humility incarnate. If he was devoted to anyone he was devoted to last· In Trinity he

was referred to as "Dear Jam" He took delight in discussing politics though he abhored indulging in the same for personal ends. According to Hardy he was a keen philosopher and an ardent politician of a pacifist and ultra radical type". But above all he was a great devotee of Goddess Mathematics in the true sense. Undaunted by poverty, depression of not getting success at F. A. examination, insistance on the part of his father to leave doing Mathematics he carried on his work on Magic Squares, Continued fractions, Hyper-geometric series, Theory of Numbers, Partition of Numbers, Elliptic functions, Fractional differentiation etc. He hated wars and refused to solve problems connected with war. So much so that he started abhoring applied Mathematics.

Any way being undernourished because of his orthodox nature and having food habits entirely unsuited to cold climate of England his health declined. He caught pulmonary tuberclosis. His worries for his parents and wife unnerved him and had a ruining effect on his health. So on Feb. 17, 1919 he had to leave England aboard S. S. Nagoya. He was to Chetput for convalseing. But as he prohesied to his wife that at Chetput every thing (including his death) would be "Chatput" he breathed his last on April 26, 1920, at the age of 32 years 4 months and 4 days only.

On January 20, 1920 from his sick bed Ramanujan wrote to Hardy in all humility "I am extremely sorry for not writing you a single letter uptill now. I have discovered a very interesting function which I call Mock-I function". He was perhaps mocking also at the mockery of Maya.

He has gone but his Mock Theta function and Roger Ramanujan identity remain and so do his Note Books As one of his biographers Sri Suresh Ram writes. "There can be no greater tribute or memorial to any genious than his work being devoutly taken up by those coming after him". Let us remember him by doing and practising mathematics,

Proc. VP., Vol. 1, 1989

SOME EXAMPLES IN ALGEBRA

By

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1 INTRODUCTION

Ganit is not only the science of *ganana*, but is also the study of *gana* (*smooh* or sets). That is why some people define mathematics as the study of sets and structures on sets. This is more aptly true about algebra where we study concepts like groups, rings and fields. These are all sets together with some structures on them. A structure is made up with the help of certain definitions and axioms. It is then given a proper shape with the help of theorems and results that are proved for these structures. In between, there lie examples. These examples are used to illustrate definitions and theorems. However, to demonstrate the limitations of a theorem, we need counterexamples. In the present talk we will discuss some new examples, and counterexamples in some algebraic structures like groups, rings and fields.

2. GROUPS :

We begin with $(Z, +)$, the additive group of integers, It is perhaps the simplest ezample of a group that one finds in an abstract algebra book. We may be little unorthodox and define 'sum' of a and b by translating $a + b$ by 1. In other works, we define an operation $\oplus$ on Z by setting

$$a \oplus b = a + b + 1 \text{ for all } a, b \in Z$$

It is quite easy now to verify the various group axioms. Here -1 is the identity (neutral) element and $b = a-2$ is the inverse of a in Z.

Going into the verification of the group axioms one finds quite interestingly that here 1 plays no special a role . Any integer positive or negative would have worked . Thus if $n \in Z$, and if $a \oplus b$ is defined by setting

$$a \oplus b = a + b + n, \text{ for all } a, b \in Z$$

then $(Z, \oplus)$ is a group . Here the identity element is $-n$ and the inverse of a is $b = a - 2n$. By giving different values to n we may get as many examplas as we like . Thus from the group $(Z, +)$, we get infinitely many examples of a group .

We may apply the technique used in the preceding paragraph to multiplicative groups also . Let, for example, $(Q_0 ,)$ denote the multiplicative group of non-zero rational numbers Let q be a fixed non-zero rational number, and led $\odot$ be defined by setting $a \odot b = abq$. One can easily verify that $(Q_0 , \odot]$ is a group Here q^{-1} is the identity element, and $b = (aq^2)^{-1}$ is the inverse of a. Here again we get, gixen one example, infinitely many examples of a group.

In the example $(Z, \oplus)$ we need, for the verification of the group axioms, not only the fact that the operation $+$ satisfies these group axioms, but also the fact that it is a commutative operation . Similarly to verify that $(Q_0, \odot)$ is a group, we use the fact (and nothing more) that Q_0 is an *abeliau* group under multiplication of rational numbers . Using this observation we may generalize the previous examples further . Suppose $(G, +)$ *is an abelian group . Let g be a fixed element of G . If we define* $\oplus$ by *setting*

$$a \oplus b = a + b + g \text{ for } a, b \text{ in } G,$$

then $(G, \oplus)$ *is an abelian group* . The inverse $-g$ *of* g *in* $(G, +)$

is the identity element in ($G, \oplus$) . Also $-(a+2g)$, the inverse of ($a+2g$) in ($G, +$), is the inverse of a in ($G, \oplus$) .

Among the simplest examples of non-commutative operations are the subtraction of numbers and the division of numbers . However, none of the sets of numbers (integers, rationals, reals or complex) is a group with respect to any of these two operations. When we search for the examples of a non-abelian group, we find these in the set of (non-singular) matrices, in the set of permutations or in the set of transformations or mappings. This raises a very natural question : Do we have a non-abelian group of numbers ? The answer is yes, infact the examples are quite simple .

Consider our familiar set Z of integers. We define a new 'addition' $\oplus$ by setting

$$a \oplus b = a + b, \text{ if a is an even integer,}$$

$$= a - b \text{ if a is an odd integer.}$$

It is obvious that Z is closed with respect to this new addition . In fact, it is only the associativity here which needs some detailed verification, the rest of the axioms are quite easy to verify . The integer zero is obviously the neutral or identity element . Also if a is an even integer then its inverse is $-a$, the negative of a, whereas if a is odd, then it is its own inverse . ($Z, \oplus$) is obviously a non-abelian group . This group has many interesting properties . For example, it is an infinite group containing infinitely many elements of finite order (each odd integer is an element of order 2) . Similarly, if H is a non-trivial subgroup of ($Z, \oplus$), then H is either an infinite group or else contains only two elements . Incidently ($Z, \oplus$) also happens to be a non-abelian group all (non-trivial) subgroups of which are abelian .

If we define a new multiplication $\odot$ on Q_0, the set of non-zero rational numbers by setting $a \odot q = ab$ if a is positive, and $a \odot b = ab^{-1}$, if a is negative. then we can show that (Q_0, $\odot$) is a non-abelian group . Similarly, the set R_0 of non-zero real numbers is also a non-abelian group with respect to $\odot$.

The concept of identity or neutral element is very important in the study of an algebraic structure . When we introduce it for the first time to our students, we do not forget to emphasize that the identity element may be different from 1 (the usual multiplicative identity) even if the set under consideration consists of numbers and the operation is the multiplication . The most common examples of multiplicative groups are (Q_0, $\odot$), (R_0, $\odot$), (Co, $\odot$), ($Z\ (p) = \{ 1, 2 ,..., p-1 \}$, $\odot_p$) for a prime p, and the group $E\ (m) = \{x \epsilon\ N : x < m, (x, m) = 1 \}$, $\odot_m$) for < a positive integer m . Here $\odot_n$ stands for the operation 'multiplication modulo n'. In all these examples the identity element is 1. Some books discuss examples like ({ 1, 4, 7, 13 }, $\odot_{15}$), where again the identity is 1. Some students, therefore, wonder whether there are examples of multiplicative groups of numbers where the identity is different from 1. Fortunately, examples as simple as ({2, 4, 8 }, $\odot_{14}$) are sufficient to make life easy for him as the element 1 is altogether absent here and the identity element is 8. In this example, we may write the set as $\{ 2, 2^2, 2^3 \}$, and 14 as $2+2^2+2^3$. One can prove that if p is a prime and n is any positive integer, then ($\{ p, p^2 , \ldots , p^n \}$. $\odot_x$) is a group, where $x = p + p^2 + \ldots + p^n$, The identity element of this group is p^n .

We mentioned earlier the group ($Z\,(p)$, $\odot_p$), where $Z\,(p) = \{1,2, \ldots p-1\}$, $\odot_p$ is the operation 'multiplication modulo p' , and p is a prime . It is one of the first examples that one is exposed to in a course on abstract algebra. Another example is the well known group

$E(n) = \{m \in N : m < n . (m, n) = 1\}$ with $\odot_n$ as the group operation, The order of this group is $\phi(n)$, where ϕ is the Euler's 'ϕ-function', and n is any fixed positive integer. A generalization of $Z(p)$ is the group $2Z(p)$, where the set $2Z(p) = \{2, 4, 6, \ldots, 2(p-1)\}$ and the operation is the multiplication modulo $2p$.

We obtained $(2Z(p), \odot_{2p})$ from $(Z(p), \odot_p)$ by multiplying elements of $Z(p)$ by 2 and also by replacing p of $\odot_p$ by $2p$. We may consider $(qZ(p), \odot_{qp})$, where q is any prime less than p. We can show that $qZ(p) = \{q, 2q, \ldots, q(p-1)\}$ is a group with respect to multiplication modulo qp.

Before we generalize the above example further, let us consider a generalization of $(E(n), \odot_n)$. As $(2Z(p), \odot_{2p})$ is a group one may guess that $(2E(n), \odot_{2n})$ is also a group for every n. This, however, is false. For if we take $n = 10$, then $E(10) = \{1, 3, 7, 9\}$ and therefore, $2E(10) = \{2, 6, 14, 18\}$. This set, however, is not closed with respect to multiplication modulo 20. Shall we now put some restriction on n? The n for which our guess fails is even. Why then not try with odd n? Let us consider $S = 2E(n) = \{2m : m \in N, m < n, (m, n) = 1\}$ for some positive odd integer n, and consider the operation 'multiplication modulo $2n$', which we denote by $*$. It can be proved that $(S, *)$ is a group.

The example that we are going to discuss now is a simultaneous generalization of the example of the preceding paragraph and the example preceding that. In the group $(2E(n), \odot_{2n})$ of the previous example, 2 is a prime and also $(2,n) = 1$. We may therefore, like to consider the set $qE(n)$, where q is a prime, and also relatively prime to n. Also in the example preceding the last one, we considered $q \leqslant (p)$, where both q and p were prime. Here also we may like to go from

$q\,Z(p)$ to $qE(n)$ for the same reasons as those for which we moved from $Z(p)$ to $E(n)$. However, for some combination of q and n, $qE(n)$ may pose the same type of problem (viz. the closure property) as we faced with $2E(n)$, when n was even. The difficulty there was avoided by choosing n to be an odd integer, Here that alone may not work, For example, if $q=3$ and $n=9$. then $qE(n)$ is not closed for $\odot_{qu}$. The fact that n is odd can also be expressed (in relation to 2) by saying that it is relatively prime to 2. This suggests that we may try with $qE(n)$ where q is not only a prime, but q and n are relatively prime too. Thus both the considerations lead us to the set $S=qE(n)$ $=\{ qm : m \lhd n, (m,n)=1 \}$ where q is a prime. $q < n$ and $(q, n) = 1$. We can show that S is a group with 'multiplication modulo qn'.

As final generalization, we consider the set $m\,E(n) = \{mx\}: x < n$, $(x, n) = 1 \}$, where $m < n$ and $(m\,n) = 1$. and the operation multipli cation modulo mn, It can be proved that (m $E(n)$, $\odot_{mn}$) is indeed a group.

3. RINGS :

We start with the following problem taken from the second edition of Herstein's *Topics in Algebra* (problem No. 11 on page 136) :

Let (R, +,) be a ring with unit element. Using its elements we define a ring (R, $\oplus_1$, $\otimes_1$) by defining .

$$a \oplus_1 b = a + b + 1, \text{ and } a \otimes_1 b = a + b + ab,$$

where $a, b \in R$ and where the addition and multiplication on the right hand side of these relations are those of $(R, +,)$.

(a) Prove that $(R, \oplus_1, \otimes_1)$ is a ring.

(b) What acts as the zero-element of (R, $\oplus_1$, $\otimes_1$) ?

(c) What acts as the unit element of (R, $\oplus_1$, $\otimes_1$)?

(d) Prove that (R, $\oplus_1$, $\oplus_1$) is isomorphic to (R, $+$,) .

(In the matter of notations in the above problem we have departed from Herstein, but it is done with a purpose) .

The above mentioned problem provides a method to construct from a special ring (viz. a ring with unit element) another ring of the *same* type .

We apply the above methed to the ring (Z, $+$,) of all integers which is a ring with unit element, we get another ring of integers with unit element zero. But what about ($2Z$, $+$,), the ring of even integers ? It is a ring without the unit element . Is it possible to apply some modified form of the above method to ($2Z$, $+$,) to get another ring of even integers ? Since 1 is not an even integer, $\oplus_1$ defined above fails even in the closure property. We therfore, modify it and define

$$a \oplus_2 b = a + b + 2 \text{ for all } a,\ b \text{ in } (2Z, +,) .$$

It is easy to see that ($2Z$, $\oplus_2$) is an abelian group with -2 as the neutral element Thus -2 is the zero element of our proposed ring . If we retain $\otimes_1$ as such, we find that it does not distri-bute over $\oplus_2$. In fact

$$a \otimes_1 (b \oplus_2 c) = ab + ac + 2a + a + b + c + 2, \text{ where as}$$

$$(a \otimes_1 b) \oplus_2 (a \otimes_1 c) = ab + a + b + ac + a + c + 2.$$

Thus, for example, if $a = b = c — 2$. then $a \otimes_1 (b, \oplus_2 c)$ is 20 whore as the value of ($a \otimes_1 b$) $\oplus_2$ ($a \otimes_1 c$) is just 11. We, therefore, have to modify $\otimes_1$ too. We let

$$a \otimes_2 b = a + b + ab/2 \text{ for all } a, b \text{ in } (2Z, +,).$$

Since a and b are even integers, $2Z$ is closed for $\otimes_2$. The associativity of $\otimes_2$ and the two distributive laws $\otimes_2$ and $\oplus_2$ can be verified easily. Thus $(2Z, \oplus, \otimes_2)$ is a ring. Its zero element is -2 and the unit elemeut is 0. Note that this time we started with a ring without unit element, bnt what we get is a ring with unit elcment. The new ring is, theeefore, a richer ring. Obviously the two rings ase different property wise.

$(2Z, +,)$ is not the only ring of integers which is without a unit element. In fact for each integer n, $(nZ, +,)$ is a ring (consisting of all multiples of n) and it is without a unite element iff $|n| > 1$. The above method can be applied to all such rings (without a unit element) to get rings with unit element. In nZ we defiue $\oplus_n$ and $\otimes_n$ by setting, for all ab in nZ

$$a \oplus_n b = a + b + n, \text{ and } a \otimes_n b = a + b + ab/n.$$

It is easy tö see that $(nZ, \oplus_u \otimes_n)$ is a ring, in which $-n$ acts as the zero-element and the integer zero works as the unit element. Further, for $|n| > 1$, the rings $(nZ, \oplus_n, \otimes_n)$ and $(nZ, +,)$ are not isomorphic.

For $n = -1$, the ring $(nZ, \oplus_n, \otimes_n)$ is $(Z, \oplus_{-1}, \otimes_{-1})$ where

$$a \oplus_{-1} b = a+b-1 \quad \text{and } a \otimes_{-1} b = a+b-ab.$$

However, $a+b-1$ and $a+b-ab$ are valid expressions in *any* ring with unit element. Therefore, if $(R, +, \otimes)$ is a ring with uuit element 1, then $(R. \oplus_{-1}, \otimes_{-1})$ is also a ring with unit element. Its zero element is 1 and the unit etement is 0, the zero element of $(R, +,)$.

New given a ring $(R, +,)$ with unit element, we can get two more rings $(R, \oplus_1, \otimes_1)$ and $(R, \oplus_{-1}, \otimes_{-1})$ both with unit elements,

the unit elements in the two rings being the zero of $(R, +,)$. We want to generalize this method. Note the special natare of these 'generating' elements 1 and -1. In a ring with a unit element, the two elements are invertible, in fact self-invertible. This suggests a possible generalization of the above construction as follows :

Let $(R, +,)$ be a ring with unit element. Suppose R contains a self invertible element g *which commutes with every element* of R. We define $\oplus_g$ and $\otimes_g$ by setting for all a, b in R

$$a \oplus_g b = a + b + g, \text{ and } a \oplus_g b = a + b + gab .$$

We can show that $(R, \oplus_g, \otimes_g)$ is a ring.

For a final ueneralization, let $(R, +,)$ be a ring with a unit element. Let g be an invertible element in R whose inverse commutes with every element of R. For a, b In R, we set

$$a \oplus_g b = a + b + g, \text{ and } a \otimes_g b = a + b + g^{-1} ab .$$

Here again it is easy to see thet $(R, \oplus_g, \otimes_g)$ is ring with nnit element.

If in the original problem $(R, +,)$ happens to be a commutatixe ring then $\otimes_1$ is also a commutative operation and henee $(R, \oplus_1, \otimes_1)$ is a commutative ring. The same is true about the ring $(R, \oplus_g, \otimes_g)$. We now ask a more specific question.

Is $(R, \oplus_g, \otimes_g)$ an integral domain if $(R, +,)$ is so?

The same question can be asked for skew fields *i.e.*, division rings) and fields. Fortunately the answer to these questions is yes. We can show that if the ring $(R, +,)$ is a commutative ring (integral domain, skew field or a field), then so is $(R, \oplus_g, \otimes_g)$.

As a final generalization we can show that if (R, $+$, $\cdot$) is a field and if n is a positive integer, then (R, $\oplus_1^n$, $\otimes_1^n$), where

$$a \oplus_1^n b = a + b + n, \text{ and}$$

$$a \otimes_1^n b = na + nb + n(n-1) + ab,$$

is also a field. The zero-element of the field is $-n$, whereas $-n+1$ is tbe uuit element.

4. FINITE FIELDS

With finite fields we have more interesting situations. In addition to the fields obtained by using the methods discussed in the earlier section, we can generate here more fields of more interesting nature, We begin with $J_p = \{0, 1, 2, \ldots, p-1\}$. If p is a prime then (J_p, $\oplus_p$, $\otimes_p$) is a field (in what follows $\oplus_x$ and $\otimes_x$ will denote respectively the *addition modulo* x and the *multiplication modulo* x). First we show that certain composite numbers too can be used to generate fields. Consider, for example, $2J_p = \{0, 2, 4, \ldots, 2(p-1)\}$ for a prime $p \geqslant 3$. We know that ($2J_p$, $\oplus_{2p}$) is an abelian group and that the even integer $p+1$ is identity element of this group. Since the two operations $\oplus_{2p}$ and $\otimes_{2p}$ are addition and multiplication of numbers, the two distributive laws hold. Thus ($2J_p$, $\oplus_{2p}$ $\otimes_{2p}$) is a fietd with 0 as the zero element and $p+1$ as the unit element.

In the above generalization of (J_p, $\oplus_p$, $\otimes_p$) to ($2J_p$, $\oplus_{2p}$, $\otimes_{2p}$) one speciality of the number 2, that we use is that it is a prime, This suggests the possibility of (qJ_p, $\oplus_{qp}$, $\otimes_{qp}$) being a field where q is also a prime (distinct form p). This indeed is the case. It can be proved that it is a field.

Similarly, if p be a prime, and if m be an integer relatively prime to p, then the system $(mJ_p, \oplus_{mp}, \otimes_{mp})$ is an integral domain and hence a field.

We close this account with rather unusual examples. In these examples the number zero is altogether absent and both the zero element and the unit element are greater than 1.

1. Let p and q be two primes (not necessarily distinct).

   Let $F = \{p, p^2, \ldots, p^q\}$, and let $x = p + p^2 + \ldots + p^q$.

   For p^i, p^j in F, we set

   $$p^i \oplus p^j = p^r, \text{ where } p^r \equiv p^{i+j} \text{ (modulo } x \text{), and}$$

   $$p^i \otimes p^j = p^s, \text{ where } p^s \equiv p^{ij} \text{ (modulo } x \text{).}$$

We can show that $(F, \oplus, \otimes)$ is a field, that p^q is its zero element and that p is the unit element.

2. Let p and q be primes and let $F = \{p^2, p^4, \ldots, p^{2q}\}$.

   Let $x = p^2 + p^4 + \ldots + p^{2q}$. If we set

   $$p^i \oplus p^j = p^r, \text{ where } p^r \equiv p^{i+j} \text{ (modulo } x \text{), and}$$

   $$p^i \otimes p^j = p^s, \text{ where } p^s \equiv p^{ij} \text{ (modulo } x \text{),}$$

then $(F_1 \oplus, \otimes)$ is a field with d^{2q} as the zero-element and p^2 as its unit element.

3. Let p, q and r be primes and $b \neq r$. Let $F = \{p^r, p^{qp}, \ldots, p^{qr}\}$ and let $x = p^r + p^{2r} + \ldots + p^{qr}$. If for $p^i, p^j F$ we set $\oplus$ and $\otimes$ as defined above, then $(F, \oplus, \otimes)$ is a field. Its unit element is p^r and the zero-element is p^{qr},

4. Let p and q be primes and let m be a positive integer relatively prime to q Let $F = \{ p^m, p^{2m}, \dots, p^{qm} \}$, and

let $x = p^m + p^{2m} + \dots + p^{qm}$. Again we set $\oplus$ and $\otimes$ as in Example 1 above. Then $(F, \oplus, \otimes)$ is a field with p^m as the unit element and p^{qm} as the zero-element.

By giving different values to the integers p, q, r and m in the above examples, we can generate infinitely many (finite) fields with zero element different from the usual zero and the unit element different from the integer 1

REFERENCES

[1] I. N. Herstein : *Topics in Algebra* (Second Edition) Vikas Publishing House 1975.

[2] Sunder Lal : Examples - New from Old - 1 *Mathematics Student* (to appear).

[3] Sunder Lal : Examples - New from Old - II *Mathematics Student* (to appear).

[4] Sunder Lal : Examples - New from Old - III *Mathematics Student* (to appear),

A BINOMIAL ANALOGUE OF THE CLASS OF ADDITION THEOREMS OF SRIVASTAVA, LAVOIE AND TREMBLAY AND ITS APPLICATIONS

By

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1. INTRODUCTION

Srivastava [5] presented certain interesting classes of generating functions in the forms :

$$\sum_{n=0}^{\infty} f_n^{\alpha+\lambda n}(x+ny)\frac{t^n}{n!}$$

and

$$\sum_{n=0}^{\infty} g_n^{(\alpha_1+\lambda_1 n, \ldots, \alpha_s+\lambda_s n)}(x_1+ny_1, \ldots, x_s+ny_r)\frac{t^n}{n!}$$

where

$$\left\{f_n^{\alpha}(x)\right\}_{n=0}^{\infty} \text{ and } \left\{g_n^{(\alpha_1, \ldots, \alpha_z)}(x_1, \ldots, x_s)\right\}_{n=0}^{\infty}$$

are general one and many-parameter sequences of functions. Further Srivastava, Lavoie and Tremblay [6] derived some general addition formulas for analogous sequences of functions and gave a number of interesting applications of the main results . Recently . Chandel and Sahgal [4] also gave a number of interesting applications and extensions of the addition theorems of Srivastava, Lavoie and Tremblay [6] .

In the present talk, a binomial analogue of the class of addition theorems of Srivastava, Lavoie and Tremblay [6] is discussed and a number of interesting applications of the main results are given .

2. Theorem.

Let the functions $B(z)$ and $z^{-1}C(z)$ be analytic in the neighbourhood of the origin, and assume (for the sake of simplicity) that

(2.1) $B(0) = C'(0) = 1$.

Define the sequence of functions $\left\{ G_n^{(\alpha,\beta)}(x) \right\}_{n=0}^{\infty}$ by means of

$$(2.2) \quad [B(z)]^{\alpha} [1 - x\, C(z)]^{-\beta} = \sum_{n=0}^{\infty} G_n^{(\alpha,\beta)}(x) \frac{z^n}{k!},$$

where α, β and x are arbitrary complex numbers independent of z. Then for arbitrary parameters λ and μ independent of z,

$$(2.3) \quad G_n^{(\alpha+\lambda\gamma,\, \beta+\mu\gamma)}(x) = \sum_{k=0}^{n} \binom{n}{k} \left(\frac{n+\gamma}{k+\gamma}\right) G_k^{(\alpha-\lambda k,\, \beta-\mu k)}(x)$$

$$G_{n-k}^{(\lambda(k+\gamma),\, \mu(k+\gamma))}(x),$$

provided that $\mathrm{Re}(\gamma) > 0$,

3. Applications of the Theorem

There are several applications of the theorem but for brevity, here we give only few examples.

Recently Chandel and Bhargava [1,2] have studied the polynomials which for $s=1$, are defined as

$$(1-z)^{-c}\left[1 - \frac{xz}{(1-z)^r}\right]^{-b} = \sum_{n=0}^{\infty} n!\ \Gamma_n^{(b,\, c)}(x, r, 1) \frac{z^n}{n!}.$$

Therefore, for $\alpha = c$, $\beta = b$, $B(z) = (1-z)^{-1}$ and $C(z) = z/(1-z)^r$, the above theorem gives

$$(3.1)\quad \Gamma^{(b+\lambda\gamma,\, c+\mu\gamma)}(x, r, 1)$$

$$= \sum_{k=0}^{n} \binom{n+\gamma}{k+\gamma} \Gamma_k^{(b-\lambda k,\, c-\mu k)}(x, r, 1)\, \Gamma_{n-k}^{(\lambda(k+\gamma),\, \mu(k+\gamma))}(x, r, 1).$$

Chandel and Bhargava [1,2] have also studied the polynomials associated to above polynomials whose generating function for $s=1$, is given by

$$(1+z)^{-c}\,[\,1-xz\,(1+z)^{r-1}\,]^{b} = \sum_{n=0}^{\infty} A_n^{(b,c)}(x,r,1)z^n.$$

Thus for $\alpha=c$, $\beta=-b$, $B(z)=(1+z)^{-1}$ and $C(z)=z\,(1+z)^{r-1}$, an appeal to above theorem gives

$$(3.2)\quad A_n^{(b+\gamma\lambda,\, c+\mu\gamma)}(x, r, 1)$$

$$= \sum_{k=0}^{n} \binom{n+\gamma}{k+\gamma} A_k^{(b\ \lambda k,\, c-\mu k)}(x, r, 1)\, A_{n-k}^{(\lambda(k+\gamma),\, \mu(k+\gamma))}(x, r. 1).$$

For $s=1$, the polynomlals of Chandel and Bhargava [3] are defined as

$$[\,C-mxz + yz^m\,]^p \left[1 - \frac{r^r xz}{(C-mxz + yz^m)^r} \right]^{-q}$$

$$= \sum_{n=0}^{\infty} B_n^{(p,q)}(m, x, y, r, r. c)\, z^n .$$

Therefore, for $\alpha=p$, $\beta=q$, $B(z) = C-mxz + yz^m$ and

$$C(z) = \frac{r^r z}{C-mxz+yz^m}.$$

an appeal to the above theorem shows that

$$(3.3)\quad B_n^{(p+\lambda\gamma,\, q+\mu\gamma)}(m, x, y, r, 1, C)$$

$$= \sum_{k=0}^{n} \binom{n+\gamma}{k+\gamma} B_k^{(p-\lambda k,\, q-\mu k)}(m, x, y, r, 1\, C)\; B_{n-k}^{(\lambda(k+\gamma),\, \mu(k+r))}(m, x, y, r, 1, C)\,.$$

REFERENCES

[1] R. C. S. Chandel and S. K. Bhargava, Some polynomials of R. Panda and the polynomials related to them, *Bul Inst. Math. Acad. Sinica*, 7 (1979), 145-149.

[2] R. C. S. Chandel and S. K. Bhargava, A further note on the polynomials related to them, *Ranchi Math. J.*, 19 (1979), 74-80.

[3] R. C. S. Chandel and S. K. Bhargava, A class of polynomials and polynomials related to them, *Indian J. Math.*, **24** (1982) 41-48.

[4] R. C. S. Chandel and S. Sahgal, Further applications and extensions of the addition theorems of Srivastava, Lavoie and Tremblay, *Indian J. Pure Appl. Math.*, **18** (1987), 830-834

[5] H. M. Srivastava, Some generalizations of Carlitz's theorem, *Pacific J. Math.*, 85 (1979), 471-477.

[6] H. M. Srivastava, J.-L. Lavoie and R. Tremblay, A class of additon theorems, *Canad Math., Bull.*, 26 (1983), 438-445.

ABSTRACTS OF THE PAPERS PRESENTED IN THE FIRST ANNUAL CONFERENCE OF VIJNANA PARISHAD OF INDIA

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A PHYSICIST'S WAY OF CHEKING THE CORRECTNESS OF MATHEMATICAL EQUATIONS

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Our message in this article is that a physical principle known as 'dimensional analysis' can be used to check the correctness of mathematical equations (i. e. equations from algebra calculus, geometry, matrices, etc.) and appreciate their beauty.

HYDRO—MAGNETIC STABILITY OE PARALLEL SHEAR FLOWS BETWEEN TWO POROUS PLATES

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In This Paper we have discussed the hydromagnetic stability of parallel shear flows between two porous plates. Growth rate of unstable modes has been obtained. Problem for non-oscillatory modes is also discussed and different results are obtained for different cases.

A TWO PHASE MATHEMATICAL MODEL AND STATISTICAL ANALYSES FOR DRAG REDUCTION IN LAMINAR FLOW

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In this paper we have studied the effect of the presence of a very thin layer of slightly visco-elastic fluid very close to the wall analytically. The presence of this layer may be attributed either to the dissolution of the coated polymers on the wall or to the wall adherence phenomenon. It has been assumed that a slightly viscoelastic fluild is being continously and uniformly injected through the wall of the circular tube in which a purely viscous Newtonian fluid is flowing. The problem has been analysed in the two regions, one of purely viscous fluild and other of thin layer near the wall seperately alongwith suitable matching cond tions Finally, we have presented the numerical results and statistical analyses for drag reduction and total flux through a cross section for a particular pressure gradient with or without the presence of thin layer of polymer additives,

THE FACTOR 10^{27} IN HINDU NUMERATION AND THE INDETERMINANCY PRINCIPLE

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The purpose of the present paper is to put forward the espeoial role of the facter 10^{27} in tenable Hindu Numeration on the basis of Sri Ram-Charitamanasa, styled as 'The Perfect of the Vedas', and to consider it the basis of the Heisenherg Indeterminancy(or Uncertainty) Principal in describing the laws of nature. The coincidence is especially important in the discussion of the tenabilities of the law of causality and the indeterminancy of infinity, and in the better understanding by contemplation of the processes involved, leaving more play for work of the imagination.

ON THE MIXED—BOUNDARY—VALUE PROBLEM IN THEORY OF ASYMMETRIC ELASTICITY

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The stress concentration problem in asymmetric elasticity around a circular hole in an infinite elastic medium has been analysed. In classical elasticity theory, the solution of a fundamental problem with mixed boundary conditions is available. However, no solution of such problems in asymmetric theory of elasticity is available. In

this paper the fundamental problem with mixed boundary conditions which generally know as the third fundamental problem for an infinite plane with a circular hole has been solved, For this, tangential traction, normal displacemeat and rotation potential are prescribed at the boundary of the hole and tractions at infinity. Complex variable technique is applied to obtain the exdlcit general solution. As particular cases, few examples are discussed. Performing numerical calculation, a comparison of stress-distribution among asymmetric elasticity, centro-symmetric elasticity and classical elasticity has been done. It has been found that the result obtained in asymmetric case gives closer approximation to the classical case case than to the centro-symmetric case. Moreover, It has been found that the effects of stresses and couple-stresses are governed by Poisson's ratio and two parameters which are dependent on the material property of the asymmetric elastic medium.

HETEROGENEOUS POPULATION GROWTH MODEL

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The Solution of the modified Kendall's population growth model equation [1, 2] have been obtained by the outhor by taking the marriage rate as an quadratic function of time [3]. In the present paper we have considered a heterogeneous population growth with respect to the marraiage rate K when it is taken as the combination of a linear and exponential function of time. Hence we have

$$K=A_1 t + A_2 e^t + A_0$$

where A_1, A_2 and A_0 are non - negative constants. The population growth model equations are given by

$$\frac{dM}{dt} + a_1 M = (a_3 + u_1)z - (A_1 t + A_2\, e^t + A_0)$$

$$\frac{dF}{dt} + a_2 F = (a_4 + u_2)z - (A_1 t + A_2 e^t + A_0)$$

and

$$\frac{dZ}{dt} + \alpha Z = A_1 t + A_2 e^t + A_0,$$

where $\alpha = a_3 + u_4$ and M, F. Z are the number of unmarried males, unmarried females, and married couples respectively. a_1, a_2 are the death rates of unmarried males and unmarried females per unit time. Integrating the above equations and neglecting the second and higher degree terms, we get

$$Z = \frac{1}{\alpha}\left[A_1 t + \frac{\alpha A_2 e^{\alpha t}}{1+\alpha} + A_o\right] - \frac{A_1}{\alpha^2} + C\, e^{-\alpha t}$$

where C is the constant of integration given by

$$C = Z_0 - \frac{1}{\alpha}\left[A_0 - \frac{A_1}{\alpha} + \frac{\alpha A_2}{1+\alpha}\right],$$

where Z_0 being initial population of married couples. We investigate the solutions for the number of numarried males and females and the total population

$$P = M + F + 2z.$$

The necessary and sufficient conditions have been obtained when the population of married couples, unmarried males and unmarried females are independent of time. The ratio of unmarried male

population to the married male population is given by

$$\left(\frac{u_1 - a_4}{a_1} \right).$$

REFERENCES

[1] Kendall, D. G. , Stochastic model and population growth, *Proc. Royal Statistical Soc.* 1: (1949), 210-264

[2] Mishra, P. , On a population model with the marriage rate depending on time - II, *Progress of Mathematics*, 22 (1988), 51-60.

[3] Pandey, H. , Solution of a population growth model with the marriage rate depending on a quadratic function of time (under publication).

MHD COUPLE STRESS FLUID FLOW PAST A STRETCHING SHEET

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In the present study, a steady two dimensional flow of couple stress fluid past a stretching sheet under the action of magnetic field perpendicular to sheet is considered. Similarity transformation yields governing equation into ordinary non-linear equation.. By using trial solution the problem is solved, and results are expressed in terms of dimensionless couple stress parameter K, and discussions have been made.

SOME THEOREMS ON GENERATING FUNCTION

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So far a number of techniques are used to obtain generating functions. But no systematic attempt has been made to get generating functions through finite difference and shift operators. In this note an attempt has been made in this direction. We have given in all four theorems about generating functions. All the results are obtained with the help of the finite difference and shift operators. One of the theorems thus obtained is the extension of a result given earlier by Rainville These theorems as special cases includes a number of results scattered in the literature (only two or three cases are discussed for brevity).

The theorems proved are as follows :

THEOREM I. If

$$e^t \, G(xt) = \sum_{n=0}^{\infty} f_n(x) t^n \quad , \qquad G(u) = \sum_{n=0}^{\infty} d_n \, u^n$$

Then

$$F(xt) = \sum_{n=0}^{\infty} \frac{((a))_n \; f_n(x)}{((b))_n} \, {}_AF_B \left[\begin{matrix} (a)+n; -t \\ (b)+n, \end{matrix} \right] t^n$$

where, $F(u) = \sum_{n=0}^{\infty} \frac{((a))_n \, d_n}{((b))_n} u_n$, and $((a))_n$ stands for

$(a_1)_n \; , \dots , (a_A)_n$.

THEOREM 2. Let the polynomials $f_n(x)$ be difined by (2.1). Then

$$(1-t)^{-a}\ H\,(xt/(1-t),\ y\,(1-t)) =$$

$$\sum_{n=0}^{\infty} (a)_n\ f_n\ (x)\ {}_{B+2}F_D \left[\begin{matrix} -m,\ a+n,\ (b);\ y \\ (d); \end{matrix}\right] t^n \quad (2.3)$$

$$\text{where,}\quad H\,(u,v) = \sum_{n=0}^{\infty} (a)_n\ d_n\ {}_{B+2}F_D \left[\begin{matrix} -m,\ a+n,\ (b);\ v \\ (d); \end{matrix}\right] u_n.$$

THEOREM 3. If the polynomials $f_n(x)$ be difined by (2 1). Then it follows that

$$\sum_{n=0}^{\infty} f_n\,(x)\ {}_{A+1}F_B \left[\begin{matrix} -n,\ (a);\ y \\ (b); \end{matrix}\right] t^n$$

$$= \sum_{n,r=0}^{\infty} \frac{d_n\ x^n}{r!}\ {}_{A+1}F_B \left[\begin{matrix} -n-r,\ (a);\ y \\ (b); \end{matrix}\right] t^{\,n+r} \quad (2.4)$$

and

THEOREM 4. If $\{\,y_n\,(x)\,\}$ be a set of polynomials defined by

$$A\,(t)\exp\,\{-xt/(1-t)\} = \sum_{n=0}^{\infty} y_n\,(x)\ t^n,$$

$$\text{where } A\,(u) = \sum_{n=0}^{\infty} e_n\ u^n$$

Then

$$(1+t)^{-a} f\,(t/(1+t)) = \sum_{n=0}^{\infty} \frac{(a)_n\ y_n\ (x)}{(b)_n}$$

$$F^{(2)} \left[\begin{matrix} a+n{:}-;\ b+n;\ xt,\ -t \\ b+n{:}-;-; \end{matrix}\right] t^n, \quad (2.5)$$

$$\text{where}\quad f\,(u) = \sum_{n=0}^{\infty} \frac{(a)_n\ e_n\ u^n}{(b)_n}$$

VISCO-ELASTIC FLOW THROUGH POROUS MEDIUM BOUNDED BY AN OSCILLATING RIGID PLATE IN A SLIP REGIME

By

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In the present paper, we deal with the study of unsteady flow of Maxwellian fluid through porous medium near an oscillating plate with slip boundary conditions. Closed form solution for velocity profile has been obtained and the influence of time-relaxation parameter on it is discussed graphically. Numerically, we discuss the veriation of velocity with rare-faction and permeability of porous medium. Also the expression for skin-friction has been derived and related results are discussed numerically.

m-DIMENSIONAL GENERATING RELATIONS FOR ${}_BA_N\ \{(x_m), y\}$

By

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In our earlier work (To appear in Bull. Cal. Math. Soc.), we have introduced a generalized polynomial system ${}_BA_N\ \{(x_m), y\}$ of m variables with the help of generating relation including the generalized hypergeometric function ${}_pF_q$ and the Lauricella function F_A. This paper enunciates a linear and a bilateral generating relation for this polynomial system. Finally several multilinear and multilateral generating relations have been deduced as special cases of our results.

FLOW IN A ROTATING POROUS PIPE

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Navier-Stokes equations of viscous incompressible flow, in a rotating frame of reference, given by Squire (1956), have been generalised to account for the porosity of the wall, with which the flow is in contact. Assuming uniform rate of suction of the liquid at the wall an approximate solution, valid for small values of both angular velocity and suction velocity, has been determined for flow undet constant pressure gradient through a circulaı pipa.

Results of Barua (1954) have veen recovered, as a particular case, when there is no porosity. The stream lines in the central plane and the projection of the stream lines on the cross-section of the pipe are compared with those in the non-porous case. An expression for the flux has been, also obtained.

LATTICE DYNAMICS OF TRANSITION METALS

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and

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A theoretical model is used for calculating the lattice dynamical properties of hcp transition metals.

DECOMPOSITION OF CURVATURE TENSOR IN R—⊕ RECURRENT FINSLER SPACE WITH NON-SYMMETRIC CONNECTION

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The purpose of present paper is to obtain some relations under the decomposition of curvature tensor R^h_{ijk} in R-⊕ recurrent Finsler space F^* equipped with non - symmetric connection parameter.

We consider n-dimensional Finsler space $F_n{}^*$ with non-symmetric connection parameter $\Gamma^i_{jk}\ (x,\dot{x})$, that is based on non-symmetric fundamental metric tensor $g_{ij}\ (x,\dot{x}) \neq g_{ji}\ (x,\dot{x})$. We write Γ^i_{jk} as below :

$\Gamma^i_{jk} = M^i_{jk} + \frac{1}{2}\ N^i_{jk}$, where M^i_{jk} and N^i_{jk} denote symmetric and skew-symmetric parts of Γ^i_{jk}.

The covariant derivatives are defined in two ways :

$$T^{\overset{i}{+}}_{\underset{j}{+}} / k = \partial_k T^i_j - (\dot{\partial}_m T^i_j)\ \Gamma^m_{pk}\ \dot{x}^p + T^m_j\ \Gamma^i_{mk} - T^i_m\ \Gamma^m_{jk} \qquad \dots (1)$$

and

$$T^{\overset{i}{-}}_{\underset{j}{-}} / k = \partial_k T^i_j - (\dot{\partial}_m T^i_j)\ \tilde{\Gamma}^m_{pk}\ \dot{x}^p + T^m_j\ \tilde{\Gamma}^i_{mk} - T^i_m\ \tilde{\Gamma}^m_{jk} \qquad (2)$$

Some theorems and results have been established. We mention here some results which have been established

(I) $$(\underset{++}{a_{lm}} - \underset{++}{a_{ml}}) \mid n = \lambda_n (a_{lm} - a_{ml}) - \lambda_n I_{lm} + \underset{++}{I_{tm}} \mid n$$

$$+ \delta n \lambda r \, N^r{}_{ml} + a_{rn} \, N^r{}_{ml} + \lambda_r \, \underset{+\ ++}{N^r{}_{ml}} \Big| \, n \, , \tag{3}$$

where $$I_{lm} = \delta_l \lambda_m - \delta_m \lambda_l \tag{4}$$

(II) $$\underset{\underset{++}{[lm/n]}}{a} + \underset{[lm\lambda_n]}{a} + \underset{[ln\delta_m]}{a} + \underset{\underset{+}{[m/n\lambda_l]}}{\delta} + P_{lr} \, N^r{}_{nm}$$

$$+ P_{mr} \, N^r{}_{ln} + P_{nr} \, N^r{}_{ml} = 0 \, , \tag{5}$$

where

$$P_{lr} = a_{lr} + \lambda_l \delta_r \, . \tag{6}$$

(III) $$\phi_{kh} / [lmn] + (P_{lr} \, N^r{}_{nm} + P_{mr} \, N^r{}_{ln} + P_{nr} \, N^r{}_{ml}) \, \phi_{kh} = 0. \tag{7}$$

INEQUALITIES OF CERTAIN CLASSES OF ORTHOGONAL POLYNOMIALS

By

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Inequalities of general pollaczek polynomials, which generalize the random walk, ultraspherical and Coulomb potential polynomials are studied. Inequalities of basic Pollaczek polynomials are then introduced which include in its fold as special cases, contineous q-ultrasberical Laguerre and Hermite polynomials and sieved orthogonal polynomials of first and second kind. Also included in the discussions are the inequalities of birth and death process polynomials and of random walk polynomials associated with birth and death process, which give rise to inequality for Carlitz - Karlin - Mc Gregor (CKM) polynomials in the particular case.

ACCELERATION COVARIANCE IN MHD TURBULENT DUSTY FLOW

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In this paper we have considered MHD turbulent dusty flow of an incompressible, viscous fluid which is nearly isotropic and spatially homogeneous. The expression for acceleration covariance has been derived and solution has been obtained in terms of difining scalars.

AN APPLICATION OF GENERALISED MEHLER FOCK TRANSFORMATION

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The Testing Space Rp(I) for Generalized Mehler Fock Transformation and various properties of dual space $R'p\,(I)$ of $R\,p\,(I)$ have been given in a series of papers by authors. This paper contains the application of transformation in evaluating the function by solving a boundary value problem in theory of elasticity. An infinitely long aniostropic, axisymmetric cylinder has been taken as a body and the functton satisfies the Laplace equation in cylindrical coordinates. The first part of paper deals with the proof of a lemma while second part deals with the formation and solution of the problem.

VIBRATION ANALYSIS OF A NON-UNIFORM CIRCULAR PLATE SUBJECTED TO ONE DIMENSIONAL STEADY STATE TEMPERATURE DISTRIBUTION

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The effect of temperature on the vibration of solid bodies is significant especially in mechanical constructions where certain parts of the body have to operate under elevated temperatures. The interest in this direction has highly increased because of rapid developments in space technology, high speed atmospheric flight and in nuclear energy applications. The present paper aims at discussing the effect of linearly transient temperature fields on the natural frequencies of a circular plate. The frequencies, defiection and moments corresponding to the first two modes of vibratlon are computed for clamped and simply-supported edge conditions by using Frobenius method.

GRAPHICS : FROM CANVAS TO COMPUTER
(A mathematical overview)

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In this paper a brief of processes involved in transfer of any graphic primitive from real-world to computer display is given. A mathematical overview of curves and surfaces in geometric design is also presented.

A NOTE ON MHD FLOW OF AN INCOMPRESSIBLE CONDUCTING FLUID OVER AN INCLINED NON-CONDUCTING IMPERVIOUS BOUNDARY

By

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Some features of the flow of a viscous incompressible fluid over an inclined non-conducting impervious boundary with a magnetic field transverse to the flow, have been studied in this paper. It is shown that this MHD flow is self- superposable and an irrotational flow on which it is superposable is determined. some observations have been made about the vorticity end stream function of the flow by using the property of superposability and self-superposability.

SOME INTEGRALS INVOLVING THE PRODUCT OF THE DOUBLE HYPERGEOMETRIC FUNCTION AND MULTIVARIABLE H-FUNCTION

By

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Using the recent result of Srivastava and Singh [Rend. Circ. Mat. Palermo II, 32 (1983), 157-187], six general formulas (four definite integral and two countour integrals) involving the products of double hypergeometric functions and multivariable H-function have been established.

SYSTEMS ANALYSIS APPROACH TO TOWN PLANNING 'A CASE STUDY OF NATIONAL CAPITAL REGION'

By
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In the present paper, we have considered the feasibility of using systems analysis approach to determine the optimal assignment of excess population of Delhi to different towns of the National Capital Region. In the problem it is desired to maximixe the efficiency of the system at the minimum possible cost.

ON A TWO VERIABLE ANALGUE OF HERMITE PLYNOMIALS

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In the present paper authors have defined a two - variable analoge of Hermite functions, $H_{k,n}^{(\nu)}(x,y)$ by generating function :

$$e^{-(t^2+T^2)}(1+2xt+2yT)^{\nu}=\sum_{k=0}^{\infty}\sum_{n=0}^{\infty}\frac{H_{k,n}^{(\nu)}(x,y)}{k!\,n!}t^k T^n$$

This two variable analogue happens to be associated with two variable Kampe - de - Feriet's hyper geometric function. Explicit forms, recurrence relations, expansions and relations in terms of other well known classical polynomials have been obtained. Finally n-variable analogue of Hermite polynomials have also been defined.

INVERSION OF AN INTEGRAL INVOLVING GENERALISED HERMITE FUNCTION

By

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The purpose of the present paper is to obtain the inversion of ihe Integral

$$g(x) = \int_0^\infty k(x/y)\, f(y) \frac{dy}{y},\ x > 0$$

where

$$k(x) = x^a \exp(-px^r)\, H_n^{(\alpha)}(x, a, p).$$

A NOTE ON NÖRLUND MEAN OF ORTHOGONAL SERIES

By

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In the present paper we prove the following theorem for Nörlund means using the (L) condition.

Thenrem : Let (P_n) be a nonnegative monotone sequence of real numbers such that $P_n \to \infty$ as $n \to \infty$ and $np_n = O(P_n)$. If an increasing sequence of natural numbers (ν_n) satisfies the condition (L), than we have under the condition (1.2), the relation

$$S_{\nu_n}(x) - t_{\nu_n}(x) = o_x(1),$$

almost everywhere.

GENERALIZATION OF FERMAT'S LAST PROBLEM

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Fermat's Last Problem:

There exist no natural numbers a, b, c, for $n>2$ such that $a^n + b^n = c^n$ where n is any natural number.

Generalized Fermat's Problem :

There will exis no natural numbers a_1, a_2, ..., a_m, b such that

$$a_1^n + a_2^n + a_3^n + \dots + a_m^n = b^n \qquad \dots \text{(A)}$$

for $n>m$ and where m, n are natural numbers, while for $n<m$, natural numbers $a_1, a_2, \dots, a_m, b$ exist which satisfy equation (A). Few illustrations are given below:

Illustrations:

1. For $m=n=3$ we have natural numbers 3, 4, 5 and 6 such that
$$3^3 + 4^3 + 5^3 = 6^3$$

2. For $m=5$ and $n=4$ we have natural numbers 2, 2, 3, 4, 4 and 5 such that
$$2^4 + 2^4 + 3^4 + 4^4 + 4^4 = 5^4$$

3. For $m=3$ and $n=2$ we have natural numbers 1, 2, 2 and 3 such that
$$1^2 + 2^2 + 2^2 = 3^2$$

4. For $m=4$ and $n=3$ we have natural numbers 1, 1, 5, 6 and 7 such that
$$1^3 + 1^3 + 5^3 + 6^3 = 7^3$$

ON GENERATING RELATIONS OF POLYNOMIAL SET $\left\{ U^*_n (x,y) \right\}$

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In this paper a polynomial set $\left\{ U^*_n (x,y) \right\}$ has been considered by means of the generating relation

$$\sum_{n=0}^{\infty} U^{*\, b;\, \lambda;\, \lambda_1;\, \lambda_2;\, V;\, l_1;\, l_2;\, (c_r)\,;\, (\alpha_g)\,;\, (\beta_h)\,;\, (a_p,\, A_p)}_{n;\, m;\, m_1;\, m_2;\, m_3;\, m_4;\, (d_s)\,;\, (y_\phi)\,;\, (\nu_\epsilon)\,;\, (b_{q+1},\, B_{q+1})} (x,y)\, t^n$$

$$= P\, F^{r:\, g;\, h}_{s:\, \phi;\, \epsilon} \left[\begin{matrix} (c_r):(\alpha_g)\,;\,(\beta_h)\,; \\ (d_s):(y_\phi)\,;\,(\nu_\epsilon)\,; \end{matrix} \;(b x^{\lambda_1} t^{\lambda_2},\, (\lambda y^{-m} t^{m_1})\right] \times$$

$$H^{l_1,\, l_2}_{p,\, q+1} \left[-V x^{m_2} y^{m_3} t^{m_4} \middle| \begin{matrix} \{a_p,\, A_p\} \\ (b_1,\, 1),\ \{b_{q+1}.\, B_{q+1}\} \end{matrix} \right];$$

where $E^{r:\, g;\, h}_{s:\, \phi;\, \epsilon} \left[\begin{matrix} (c_r):(\alpha_g)\,;\,(\beta_h)\,; \\ (d_s):(y_\phi)\,;\,\nu_\epsilon)\,; \end{matrix} \;(b x^{\lambda_1} t^{\lambda_2}),\, \lambda y^{-m} t^{m_1})\right]$

are Appel functions of two variables represented in most modified notation due to Burchnall and Chaundy,

$H^{l_1,\, l_2}_{p,\, q+1} \left[-V x^{m_2} y^{m_3} t^{m_4} \middle| \begin{matrix} \{a_p\,,\, A_p\} \\ (b_1\,,\, 1),\quad \{b_{q+1}\,,\, B_{q+1}\} \end{matrix} \right]$

are Fox's H-function and

$$P = \frac{\prod_{j=l_2+1}^{p} \Gamma(a_j - A_j b_1) \prod_{j=l_1+1}^{q+1} \Gamma(1 - b_j + F_j b_1)}{\prod_{j=1}^{l_2} \Gamma(1 - a_j + A_j b_1) \prod_{j=2}^{l_1} \Gamma(b_j - B_j b_1)} \times (-V x^{m_2} y^{m_3} t^{m_4})^{-b_1}$$

A number of generating relations for several interesting classes of polynomials have been obtained from the main result. Some useful transformations and series identities have been used to find some useful results. Direct applications of Srivastava and Carlitz have reduced the theorems to interesting special cases. It has been observed how readily these new generating functions can be extended to help the advancement in the field of special functions.

SOME INTEGRAL REPRESENTTAIONS OF A GENERALIZED POLYNOMIAL SET $\left\{ M_n^* (x,y) \right\}$

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The present paper deals with integral representations of the polynomial set $\left\{ M_n^* (x,y) \right\}$. We have obtained finite single, infinite single and finite double integral representations for $\left\{ M_n^* (x,y) \right\}$ in terms of theorems and corollaries. Some special interesting cases have also been derived.

A GENERAL COINCIDENCE THEOREM

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A coincidence theorem generalizing Goebel's coincidence theorem Jungck contraction principle, Matkowski's fixed point theorem, Fisher's generalization of Jungck theorem and coincidence theorems of Kominek and Singh-Gairola-Mehndiratta is established for four systems of transformations on a product of n arbitrary sets with values in metric spaces

FIXED POINTS OF MULTIVALUED MAPPING IN PROBABILISTIC METRIC SPACE

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Hardwar-249494 **Kotdwar (Garhwal)**

Let $C(X)$ denote the set of all nonempty subsets of a Menger space (X, F, t) and let $P, Q: X \to X$ and $S, T: X \to C(X)$. In this paper we prove common fixed point theorems for P, Q, S and T. These results are generalizations of some well known coincidence and fixed point theorems for single-valued and multivalued mappings in metric and probabilistic metric spaces.

AN EXTENSION OF ANCIENT INDIAN CUBE-ROOT METHOD

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In this paper an attempt is made to extend Bhāskarāchārya's method of obtaining cube-root of a perfect cubic number to 5th root of any integer. The method indeed may be applied to compute n^{th} root of any positive integer.

A FIXED POINT THEOREM FOR A SEQUENCE OF MAPPINGS ON 2-METRIC SPACES

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In this paper a fixed point theorem on a 2-metric space is presented

SOME COMMON FIXED POINT THEOREMS IN 2-METRIC SHACES

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In this paper following result is established for a 2-metric space (X, d) and two self-mappings S and T :

If there exists real numbers p and q such that
$p \in (0, 1)$ and $p > 1 + q$ and

$$[d(x, Sx, a) + d(y, Ty, a)]\ b(x, Ty, a)$$
$$\leqslant q \max \{[d(x, y, a)]^2, [d(x, Sx, a)]^2, [d(y, Ty, a)]^2, (1/p)([d(x, Ty, a)]^2 + [d(y, Sx, a)]^2\}$$

where $x, y, a \in X$. Then S and T have unique common fixed point.

VARIOUS FIXED POINT THEOREMS

By

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Fixed point theory has been a topic of great interest among researchers. This theory has been of a great help to bridge the gulf between "Pure Mathematics and Applied Mathematics". Fixed point theorem given by Banach, Brouwer, Schauder, Glicksberg and Browder have important applications in Pnysics. Economics, Engineering and in some other Mathematical Sciences. these theorems along with their applications have been started here.

Definition (Fixed point) :- Let $T: X \to X$ where X is a metric space. If $x_0 \epsilon X$ be such that $Tx_0 = x_0$, than x_0 is called a fixed point of the operator T.

1. Banach's Contraction Mapping Theorem .

Let (x, d) be a complete metric space and T be an operator, mapping x into itself such that

$$d(Tx, Ty) \leqslant \alpha\, d(x, y) \quad \text{for all } x, y \in X$$

where $0 < \alpha < 1$.

Then there exists a unique fixed point of the operator T in X.

Banach's fixed point theorem has been applied to find the unique solution of linear algebraic equations, differential equations, integral equations and as well as to implicit function theorem.

2. Brouwer's Theorem :

Every continuous mapping of a closed bounded convex set in R^n into itself has a fixed point.

Brouwer's theorem has been applied to the theory of positive matrices which are important in economics, probability theory and in other fields.

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3. Schauder's Theorems.

(i) (Schauder's First Theorem) :

If S is a convex compact subset of a normed space, every continuous mapping of S into itself has a fixed point.

(ii) (Schauder's Second Theorem) :

If S is a convex closed subset of a normed space and R a relatively compact subset of S, then every continuous mapping of S into R has a fixed point.

Schauder's theorems have been applied to Swirling–flow problem, to a problem concerning nonlinear oscillations of a pendu-lum and in Quantum and Statistical Physics.

4. Fan-Glicksberg Theorem.

Let C be a non-empty compact convex subset of a normed linear space X. For $x \in C$, let $F(x)$ denote a non-empty closed convex subset of C. Assume the following property (called upper semi conti-nuity of F):

If $x \in C$ and V is an open subset of C containing $F(x)$, then there is an open subset U of C containing x such that for all $x \in U$, $F(x) \subset V$. Then there exists $a \in C$ such that $a \in F(a)$.

Set valued maps often arise in the control theory for various systems in engineering and economics, in the theory of differential games etc. Fixed point theorems, such as the one above are useful in these situations.

5. Browder's Theorem.

Let X be a uniformly convex Banach space and S, a non-empty closed bounded and convex subset of X. If Z is a family of non-expansive maps from S to S such that $FG = GF$ for all $F, G \in Z$ then Z has a common fixed point in S.

This has been proved to be of remarkable value. Browder himself has applied it to show the existence of periodic solutions of a very general class of non-linear equations of evolution in infinite dimensinal spaces.

A COMMON FIXED POINT THEOREM OF COMMUTING MAPPINGS IN SAKS SPACE

By

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Common fixed point theorem for four mappings is proved in a Saks space.

In this paper, motivated by the work of Cho and Singh [Kobe J. Math. 3 (1986), 1-6] on Saks space, we prove a fixed point theorem for four mappings on a Saks space. Our theorem extends the result of Xie Ping Ding [Math. Sem. Notes, **11** (1983), 301-305] on the fixed point theorem for cummuting mappings on the line of Jungck [Amer. Math. Monthly, 03 (1976), 261-263.

ON APPROXIMATE SOLUTIONS OF MULTIVALUED MAPPING IN ORDERED BANACH SPACES

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Let X and Y be ordered Banach spaces, $A : X \rightarrow Y$ be a linear mapping and $N : X \rightarrow Y$ a nonlinear mapping of some type. To solve problems in partial and ordinary differential equations, these problems may be described in terms of an operator equation

$$Ax + Nx = z \; (x \in X, z \in Y) \qquad \ldots(1)$$

It is now only required to solve the operator eqn. (1).

In This paper we have given some results on approximate solvability of multivalued mappings in ordered Banach spaces. The existence of non zero solution of eqn. (1) be proved. Theorem (1) is improved version of P. S. Milojevic. We have also given the more generalised versions of theorem (1) for the purpose of appropriate applications.

NORMAL STRUCTURE AND FIXED POINT THEOREMS IN CONVEX METRIC SPACE

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In this paper we have proved some fixed point theorems in convex metric space using the action normal structure. Banach space fells in the category of convex metric space. Hence here we have generalized the result of Kannan [Proc. Math Soc.38 (1973), 111-118] and Kirk , Proc. Amer. Math. Soc Symp. Pure Math. :8 (1970), 162-168.

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CONTENTS